Engineering Analysis with Boundary Elements 94 (2018) 94–102

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Domain-decomposition generalized finite difference method for stress analysis in multi-layered elastic materials

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ARTICLE INFO

Keywords:
Multi-layered materials
Meshless method
Generalized finite difference method
Domain decomposition technique
Elasticity

ABSTRACT

The generalized finite difference method (GFDM) is a relatively new meshless method for the numerical solution of certain boundary value problems. The method uses the Taylor series expansions and the moving least squares approximation to derive explicit formulae for the required partial derivatives of unknown variables. In this paper, we document the first attempt to apply the GFDM for the numerical solution of two-dimensional (2D) multi-layered elastic problems. A multi-domain GFDM scheme is proposed to model the composite (layered) elastic materials. The composite material considered is decomposed into several sub-domains and, in each sub-domain, the solution is approximated by using the GFDM-type expansion. On the subdomain interface, compatibility of displacements and equilibrium of tractions are imposed. Preliminary numerical experiments show that the introduced multi-domain GFDM is very promising for accurate and efficient numerical simulations of multi-layered materials.

1. Introduction

Following the rapid improvement of industrial technology, more and more new materials have been synthesized, designed and utilized in recent years. Among these materials, the multi-layered materials which contain single or multiple layers have been widely utilized in industrial application to improve machining performance [1–4]. The coating layers can protect the substrate material against adhesion diffusion and intensive abrasive wear, due to their better temperature and wear resistant properties. However, the rather complex and expensive experimental investigations on composite (layered) materials underlie a general lack of the analytical or numerical modeling efforts which can accurately and efficiently predict the performances of multi-layered coating structures [5–8].

The well-established and widely applied finite element method (FEM) offers without doubt many advantages in solving multi-layered problems due to its flexibilities in dealing with the geometry, loading type and nonlinearities of the coating layers. The FEM itself, however, has also many inherent shortcomings especially when a re-meshing is required or when the elements become highly distorted [9,10]. As an alternative numerical approach, the boundary element method (BEM) can be applied efficiently to avoid such shortcomings become of the boundary-only discretization and its semi-analytical nature. In the past two decades, the BEM has been rapidly improved and can be nowadays considered as a competing method to the FEM. As a price paid for such a merit, the classical BEM, however, has to compute various singular and/or nearly singular integrals over the boundary elements, which is usually a cumbersome and non-trivial task [11–17].

Over the past two decades, some considerable effort was devoted to circumventing the shortcomings associated with the classical FEM and BEM methods. This drives to the development of various meshless methods which require neither domain nor boundary meshing [18–20]. The meshless methods still require discretization via sets of boundary and/or domain nodes, but these nodes need not have any connectivity and the trial functions are built entirely in terms these scattered irregular clouds of nodes. For an overview of the state of the art, we refer the readers to Refs. [16,21–25], as well as the references therein.

The generalized finite difference method (GFDM) is a relatively new meshless method. The main idea of the method is to combine the Taylor series expansion and the moving-least squares (MLS) approximation to derive explicit formulae for the required partial derivatives of unknown variables. The derivatives of unknown variables, and then, can be approximated by a linear combination of function values with respect to its neighboring nodes. The key idea of the GFDM was proposed in the early eighties by Lizska and Orkisz [26,27] and were later essentially extended and improved by many other authors [28–37]. Prior to this study, this

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https://doi.org/10.1016/j.enganabound.2018.06.006
Received 14 April 2018; Received in revised form 10 June 2018; Accepted 10 June 2018

method has been successfully tried for 2D and 3D parabolic and hyperbolic equations [30,38,39], third- and fourth-order partial differential equations [40], dynamic analysis of beams and plates [41], non-linear elliptic partial differential equations [42], and applied inverse problems [31,43,44]. In recent years, several other meshless methods have been considered and developed by researchers to obtain numerical solutions for different types of partial differential equations. The methods include, but are not limited to, the element-free Galerkin (EFG) method [45], the local radial point interpolation method (LRPIM) [46], the meshless local Petrov-Galerkin (MLPG) method [47], the boundary point interpolation method (BPIM) [48], the method of fundamental solutions (MFS) [49], and the singular boundary method (SBM) [25]. Each of the above-mentioned methods has its own merits and demerits. Interested readers are referred to excellent overview articles [24,25,45] on the development of this topic.

In this paper, we extend the GFDM to problems of stress analysis in multi-layered elastic materials. The multi-layered problems under consideration in this paper are solved using a non-overlapping domain decomposition method (DDM), in which the composite material is decomposed into several sub-domains and, in each sub-domain, the solution is approximated by using the GFDM-type expansion. On the subdomain interface, compatibility of displacements and equilibrium of tractions are imposed. These interface continuity conditions are satisfied in a least-squares sense in the same way as the boundary conditions of the problem. There are two main forms of the DDMs, which are the overlapping DDM and non-overlapping DDM. As compared to overlapping methods, the non-overlapping DDM has become very appalling for its inherent parallelism and flexibility. We refer to the papers [50,51] for theoretical and numerical results for non-overlapping DDM and to the references given there. In recent decades, the combination of the DDM and other methods has been proposed for the numerical solutions of elastic problems in layered materials. In Ref. [52], Berger and Karageorghis used the method of fundamental solutions (MFS) to deal with the stress analysis in layered elastic materials. In Ref. [53], Gu et al. applied the meshless singular boundary method (SBM) in conjunction with domain decomposition technique for the stress analysis of layered elastic materials. In Ref. [54], Yan et al. employed a local RBF collocation method for solving elastic waves in multi-layered functionally graded materials.

A brief outline of the rest of the paper is organized as follows. In Section 2, the GFDM formulation and its numerical implementation for the solution of general 2D elastic problems are briefly discussed. A multi-domain GFDM scheme for the solution of multi-layered elastic problems is presented in Section 3. In Section 4, three benchmark numerical examples are presented to validate the computational code and assess the performances of the proposed GFDM scheme. Finally, some conclusions and remarks are provided in Section 5.

2. The GFDM for isotropic problems in linear elasticity

2.1. Statement of the basic problem

The equilibrium equations for 2D problems in linear elasticity, in terms of the displacements, $u_i(\boldsymbol{x})$, $i=1, 2$, can be stated as [4]

$$\left\{2\frac{1-\mu}{1-2\mu}\right\}\frac{\partial^2 u_1(\boldsymbol{x})}{\partial x_1^2}+\frac{\partial^2 u_1(\boldsymbol{x})}{\partial x_2^2}+\left\{\frac{1}{1-2\mu}\right\}\frac{\partial^2 u_2(\boldsymbol{x})}{\partial x_1\partial x_2}=f_1(\boldsymbol{x}), \tag{1}$$

$$\left\{\frac{1}{1-2\mu}\right\}\frac{\partial^2 u_1(\boldsymbol{x})}{\partial x_1\partial x_2}+\frac{\partial^2 u_2(\boldsymbol{x})}{\partial x_1^2}+\left\{2\frac{1-\mu}{1-2\mu}\right\}\frac{\partial^2 u_2(\boldsymbol{x})}{\partial x_2^2}=f_2(\boldsymbol{x}), \tag{2}$$

subject to the boundary conditions

$$u_i(\boldsymbol{x})=\bar{u}_i(\boldsymbol{x}) \quad \boldsymbol{x}\in\Gamma_u \quad (\textit{Dirichlet boundary conditions}), \tag{3}$$

$$t_i(\boldsymbol{x})=\sigma_{ij}(\boldsymbol{x})n_j(\boldsymbol{x})=\bar{t}_i(\boldsymbol{x}) \quad \boldsymbol{x}\in\Gamma_t \quad (\textit{Neumann boundary conditions}), \tag{4}$$

where $\boldsymbol{x}=(x_1, x_2)$, μ is the Poisson's ratio, $t_i(\boldsymbol{x})$ denotes the component of boundary traction in the ith coordinate direction, $n_j(\boldsymbol{x})$ are the outward unit normal vector, Γ_u and Γ_t construct the whole boundary of the domain Ω, $\bar{u}_i$ and $\bar{t}_i$ represent the prescribed displacements and tractions, respectively, $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ in Eqs. (1) and (2) denote the inhomogeneous terms. Here and in the following, the customary Einstein's notation for summation over repeated subscripts is applied. The kinematics of deformation is described by the linear strain tensor

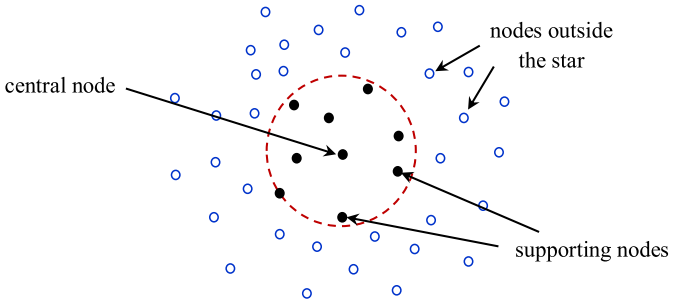

Fig. 1. An irregular cloud of points and the selection of stars in the GFDM.

$$\varepsilon_{ij}(\boldsymbol{x})=\frac{1}{2}\left\{\frac{\partial u_i(\boldsymbol{x})}{\partial x_j}+\frac{\partial u_j(\boldsymbol{x})}{\partial x_i}\right\}, \tag{5}$$

where sufficiently small displacements and displacement gradients are assumed. The stresses $\sigma_{ij}(\boldsymbol{x})$ are related to the strains $\varepsilon_{ij}(\boldsymbol{x})$ through generalized Hooke's law by

$$\sigma_{ij}(\boldsymbol{x})=2G\left(\varepsilon_{ij}(\boldsymbol{x})+\frac{\mu}{1-2\mu}\varepsilon_{kk}(\boldsymbol{x})\delta_{ij}\right), \tag{6}$$

where G stands for the shear modulus, δ_{ij} is the well-known Kronecker delta. The boundary tractions $t_i(\boldsymbol{x})$, $i=1, 2$, are defined in terms of the stresses as

$$t_i(\boldsymbol{x})=\sigma_{ij}(\boldsymbol{x})n_j(\boldsymbol{x}), \quad \boldsymbol{x}\in\Gamma. \tag{7}$$

Eqs. (1)–(7) completely describe the isotropic problems in linear elasticity.

2.2. Explicit formulae in GFDM

Without loss of generality, let us consider the following general differential equation in the 2D domain [42]

$$a_1\frac{\partial U}{\partial x_1}+a_2\frac{\partial U}{\partial x_2}+a_3\frac{\partial^2 U}{\partial x_1^2}+a_4\frac{\partial^2 U}{\partial x_2^2}+a_5\frac{\partial^2 U}{\partial x_1\partial x_2}=f(\boldsymbol{x}), \tag{8}$$

or for brevity

$$L_2[U]=f(\boldsymbol{x}), \tag{9}$$

where $L_2[U]$ is a second-order partial differential operator, $f(\boldsymbol{x})$ is known function, a_i, $i=1, 2, \ldots, 5$, are constants.

In order to obtain the explicit GFDM formulae for partial differential equations, an irregular cloud of points is scattered in the computational domain (see Fig. 1). For each given node $\boldsymbol{x}_0$, named as the central node, the m nearest nodes $\boldsymbol{x}_i$ ($i=1, 2, \ldots, m$), called the neighbors or supporting nodes, will be found within a prescribed distance d_m from the central node $\boldsymbol{x}_0$, i.e., $|\boldsymbol{x}_i-\boldsymbol{x}_0|\le d_m$. The concept of the '*star*' then refers to the area of all supporting nodes in relation to the central node [38]. Each node scattered inside the computational domain has an associated star assigned.

Suppose U_0 is the value of the function at the central node $\boldsymbol{x}_0$ and U_i, $i=1, 2, \ldots, m$, are function values at the rest of the nodes inside the star. Expanding the values of U_i around the central point $\boldsymbol{x}_0$ using the Taylor series expansion, we have [39,55]

$$U_i=U_0+h_i\frac{\partial U_0}{\partial x_1}+k_i\frac{\partial U_0}{\partial x_2}+\frac{h_i^2}{2}\frac{\partial^2 U_0}{\partial x_1^2}+\frac{k_i^2}{2}\frac{\partial^2 U_0}{\partial x_2^2}+h_ik_i\frac{\partial^2 U_0}{\partial x_1\partial x_2}+\ldots\ldots, \quad i=1, 2, \ldots, m. \tag{10}$$