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High order mesh-free method for frictional contact

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ABSTRACT

This work concerns numerical simulations of the frictional contact of two elastic deformable bodies and a novel treatment technique of incompatible contact nodes. These numerical simulations are based on a high order mesh-free method coupling the Moving Least Square (MLS) with the Asymptotic Numerical Method (ANM) and on a regularization technique. The obtained nonlinear problem is solved by using ANM applied on strong formulation in order to avoid numerical integrations. According to ANM, the Taylor series development of unknown variables on nodes of nonlinear contact problem leads to a sequence of linear systems to be solved. These linear systems are then discretized by a collocation mesh-free approach by using the MLS functions and a continuation method is adopted to evaluate the solution. The contact problem is identified to boundary conditions which are replaced by force-displacement relations through a regularization technique. The ability of the proposed approach is tested on some contact elastic deformable bi-dimensional examples. The obtained results are compared to an analytical solution and to a numerical solution obtained by the Newton–Raphson method coupled also with MLS approximation.

1. Introduction

The nonlinear contact problem involving deformable bodies is one of the most important problems. It's frequently encountered in the industrial domain such as the rolling, stamping, hot and cold rolling and in the field of robotics, for example, compliant grasping [1,2], robotic assembly of snap-fits [3] and assisted surgery [4]. The mechanical contacts are also common robotic problems in agriculture, food-processing and surgical robotic systems.

The mathematical modeling of these type of problems is difficult due to the many material, geometric and contact nonlinearities and the numerical treatment of incompatible contact nodes. The knowledge of their solutions are of great importance, because they permit the good optimization of designs and lead to the improvement of the performance of these systems. The access to these solutions can only be done by numerical methods.

Efficient numerical methods are needed and are crucial for an accurate simulation of contact problems. During these last years, several numerical methods have been proposed in the literature and used to solve the industrial contact problems. One can cite the conventional mesh-based numerical methods such that the Finite Element Method (FEM) and Finite Volume Method (FVM). The FEM [5–8] is nowadays widely used by engineers in all fields and several well-assessed commercial codes are available. The applications of FEM range from structural mechanics, thermal analysis, acoustics, fluid dynamics, electromagnetism and even multi-physics (i.e. coupling of physical phenomena) simulations. Although FEM meshes provide the generality to handle complicated geometries, appropriate mesh structures are often difficult to be created or modified especially for applications where meshes must be reconstructed automatically during the computational process.

To reduce the difficulty of meshing and remeshing of complex contact problem domains, a new generation of computation methods, named as mesh-free or meshless methods are developed without defining mesh [9–13].

In FEM, the contact surfaces are discretized by segments or curves. Three main types of contact elements are: node-node [14], node-surface [15] and surface-surface [16]. Meshless methods are nowadays an attractive alternative to Finite Element Methods FEM for simulating contact problems. Their main advantage is that the basis functions are defined from a scattered set of nodes, avoiding in this way the requirement of a mesh to support them as in traditional FEM (a background integration grid, however, is required). Meshfree approximation schemes are also much less sensitive than FEM to irregular nodes distributions and entanglement, which make them more robust to deal with large deformations [17–20]. The most popular mesh-free approximants are based on the Moving Least Squares MLS method [21] which have been gaining attention. Many meshless methods are developed in Duarte and Oden [22]; node interpolation method [23–25] and Mesh-free Weak-Strong form MWS [26]. Some of the important features of meshless methods can be found in Liu [27], those that make them better to standard methods. See also a review on recent meshless methods in Nguyen et al. [28]. The collocation method is used by Nguyen et al. [12]. The majority of published works are based on meshless method with a weak formulation and a penalty method for analyzing large deformation elasto-plastic

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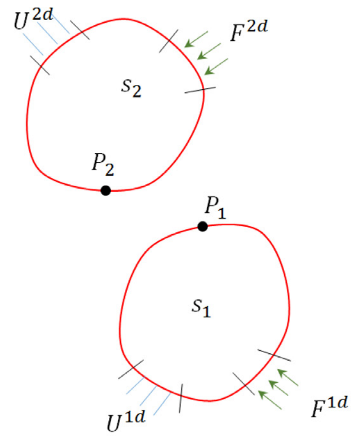
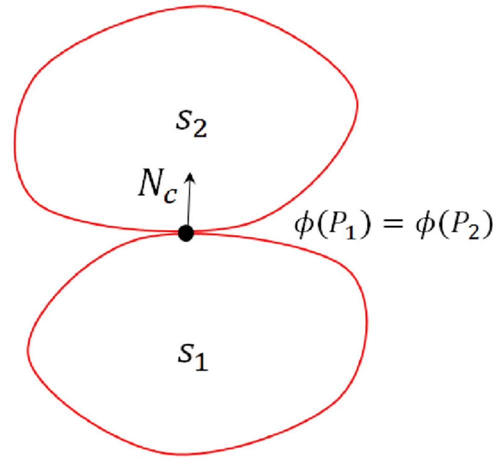


(a) Before contact



(b) During contact

Fig. 1. Finite deformation contact: before and during contact.

contact problem. The solution is computed numerically by some prediction-correction procedures. An alternative to these procedures, one find the Asymptotic Numerical Method (ANM) which is an efficient algorithm to solve the nonlinear problems [29–32].

Other authors have proposed the coupling of the Asymptotic Numerical Method (ANM) with the Moving Least Square (MLS) approximation for the resolution of the material mixing mechanical problem in the Friction Stir Welding (FSW) process [33]. In this modelling, the strong form of the equilibrium and the ANM method are used to avoid numerical integration and to reduce the number of stiffness matrix inversions respectively. In the work [34], the authors have used the same algorithm for simulating material mixing in FSW process of a 2D mechanical-thermal problem.

In a recent work, Belaasilia et al. [35] have used this coupling applied to a strong formulation for simulating elasto-plastic structures with contact in the context of large deformations. This coupling is based on the Asymptotic Numerical Method ANM which is used in the meshless collocation framework to extend its application field to elasto-plastic problems with contact of deformable and rigid foundation bodies. The efficiency of this coupling is to take into account of large deformations and to avoid the meshing distortion problem. According to ANM, the Taylor series is performed to obtain a sequence of linear systems to be solved. These linear systems are then discretized by a collocation meshless approach by using the Moving Least Squares MLS and a continuation method is adopted to evaluate the whole solution. The unilateral contact problem is identified to boundary conditions which are replaced by force-displacement relations through a regularization technique. The performance of the proposed approach is tested on several elasto-plastic bi-dimensional elastic plate and a circular rigid foundation. The obtained results are compared to those computed by the Newton–Raphson method. It is important to emphasize that in this work the contact studied concerns only a rigid-deformable bodies is considered, moreover the problem of the non-compatibility of the nodes is not treated. It is important to underline that the compatibility condition between contact nodes is not always satisfied. If the contact nodes are not compatible, it is difficult to look for and to determine these nodes that will be in contact. The use of finite elements method is confronted with a difficult implementation and requires a high level of technical.

The aim of this paper consists to use in the first time the coupling cited above for the resolution of the contact problem of deformable-deformable bodies and to propose a novel treatment technique of incompatible contact nodes based on Moving Least square approximation, and a regularization technique. In order to have a compatibility between the nodes in contact and that the condition of this compatibility is always verified, we propose virtual nodes without adding them in the computation. This technique permits firstly to make the incompatible case to a compatible case without increasing the number of nodes by adding virtual nodes and on the other hand to look for their displacements by using their coordinates.

The paper is organized as follows. In Section 2, the mechanical equilibrium of two deformable elastic bodies under a strong form and the contact modelling are presented. In Section 3, the description of the high order mesh-free method is detailed. Section 4 is devoted to the validation of the proposed approach by examining two examples of 2D elastic structures, and finally, the conclusion is given in Section 5.

2. Modelling

2.1. Mechanical equilibrium of two deformable elastic bodies

Consider two $2D$ deformable elastic bodies S^1 and S^2 in contact, which occupy the volumes Ω_1 and Ω_2 of boundaries $\partial\Omega_1$ and $\partial\Omega_2$. These bodies are subjected to external loadings F^{1d} and F^{2d} applied on boundaries $\partial\Omega_{1F}$ and $\partial\Omega_{2F}$ and to imposed displacements U^{1d} and U^{2d} on their boundaries $\partial\Omega_{1u}$ and $\partial\Omega_{2u}$ as depicted on Fig. 1. We observe that two points P_1 and P_2, in the initial configuration of the bodies which are distinct, can occupy the same position in the current configuration, $\Phi(P_1) = \Phi(P_2)$, within the deformation process. The local equilibrium equations, for each body j; $j = 1, 2$, completed by their boundary conditions are written:

$$\begin{cases} div(\sigma^j) = 0 & in \quad \Omega_j \\ \sigma^j . N^j = \lambda F^{jd} & on \quad \partial\Omega_{jF} \\ U^j = \lambda U^{jd} & on \quad \partial\Omega_{ju} \end{cases} \tag{1}$$

where σ is the Cauchy stress tensor, N^j is is the unit outward normal to boundary $\partial\Omega_{jF}$ and λ is a control parameter, U^j is the displacement, U^{jd} is an imposed displacement on the boundary $\partial\Omega_{ju}$ and F^{jd} is the stress vector applied on the boundary $\partial\Omega_{jF}$. The Cauchy stress tensor σ^j satisfy the Hooke's constitutive law given by the following equation:

$$\sigma^j = C.\epsilon^j \tag{2}$$

where C is the fourth order elastic modulus tensor, assumed to be symmetric positive definite and ϵ^j is the strain tensor given in a small strain hypothesis by:

$$\epsilon^j = \frac{1}{2}({}^T\nabla U^j + \nabla U^j) \tag{3}$$