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Dynamic Green's functions for multiple circular inclusions with imperfect interfaces using the collocation multipole method

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ABSTRACT

This paper presents a semi-analytical approach to solve anti-plane dynamic Green's functions for an elastic infinitely extended isotropic solid (matrix) containing multiple circular inclusions with imperfect interfaces. A linear spring model with vanishing thickness is employed to character the imperfect interface. The multipole expansions of anti-plane displacement of the matrix and inclusion, induced by a time-harmonic anti-plane line force located in the matrix or in the inclusion, are expanded by using Hankel and Bessel functions, respectively. The imperfect interface condition is satisfied by uniformly collocating points along the interface of each inclusion. For the imperfect interface condition, the normal derivative of the anti-plane displacement with respect to a non-local polar coordinate system is developed without any truncation error for multiply-connected domain problems. For the case of one circular inclusion, the proposed quasi-static stress field matches well with the analytical static solution. The proposed quasi-static stress fields containing two and three circular inclusions are critically compared with those calculated by static analysis using the finite element method. Finally, extensive studies are presented to investigate the effects of the frequency of excitation, imperfect interface and separation between inclusions on the dynamic Green's functions.

1. Introduction

Time-harmonic Green's functions can be applied to formulate the boundary element method for the eigenvalue problems [1,2] and can also be used to deal with Eshelby inclusion problems [3–5] and scattering problems in elastodynamics [6–8]. Extensive studies have been carried out on dynamic Green's functions for homogeneous media by using various analytical methods [9–13]. Senjuntichai and Rajapakse [9] analytically constructed dynamic Green's functions of homogeneous poroelastic half-plane problem using Fourier integral transforms. Norris [10] derived the three-dimensional dynamic Green's functions in anisotropic piezoelectric, thermoelastic and poroelastic solids by using plane wave transform method. Based on the Radon transform method, Wang and Achenbach [11] presented three-dimensional time-harmonic elastodynamic Green's functions for anisotropic solids, and Wang and Zhang [12] derived dynamic Green's functions for linear piezoelectric solids. Wang and Zhong [13] derived the two-dimensional time-harmonic Green's functions in transversely isotropic piezoelectric solids.

In contrast, investigations on dynamic Green's functions for composite materials are rarely reported in literature due to the fact that the corresponding dynamic Green's functions are mathematically quite complicated. Wang and Sudak [14] analytical derived the antiplane time-harmonic Green's functions for a circular inhomogeneity with an imperfect interface. Khojasteh et al. devised three-dimensional dynamic Green's functions in transversely isotropic bi-materials [15] and tri-materials [16]. Chen et al. [17] applied the null-field boundary integral equation method (BIEM) to derive anti-plane dynamic Green's functions for several circular inclusions with imperfect interfaces. Recently, efficient procedures based on semi-analytical expansions are developed for Helmholtz BIEs on problems involving multiple disk-shaped scatterers [18,19].

In general, the Green's function solution can be represented by two parts; one is the known singular solution that is the so-called fundamental solution or the free-space Green's function, and the other is the unknown regular part that is mainly incorporated to satisfy the specified boundary conditions for the problem under consideration. Most of Green's functions found in the literature are analytically expressed in closed or series forms. However it is not the case for some complicated problems and then Telles et al. [20] proposed the numerical Green's functions in the formulation of the boundary element method for solving the fracture mechanics. Consequently, the collocation multipole method is presented to semi-analytically solve dynamic Green's functions for an elastic infinitely extended isotropic solid (matrix) containing multiple circular inclusions with imperfect interfaces.

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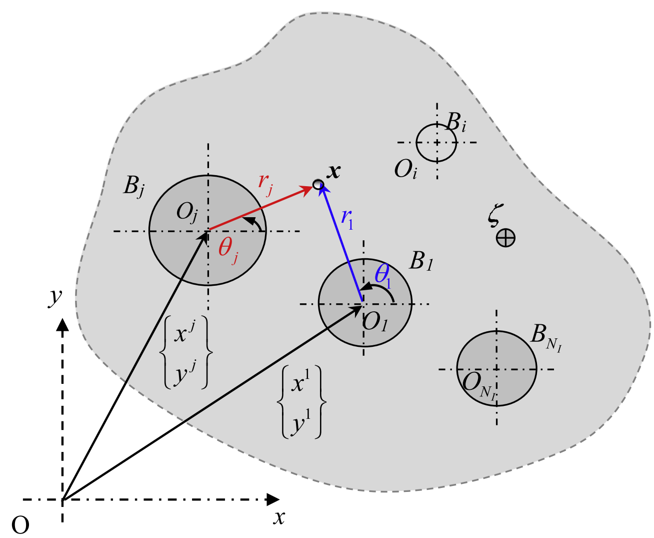

Fig. 1. An elastic infinitely extended isotropic solid (matrix) containing multiple circular inclusions with imperfect interfaces subjected to a time harmonic anti-plane line force located at ζ.

The multipole method for solving multiply-connected domain problems was first proposed by Závǐska [21], the multipole expansion being the so called the wave function expansion. The addition theorem is often employed to transform the multipole expansion into one of the local coordinate systems, attached to the center of each object such as an inclusion, to satisfy the specified boundary conditions. In the case of the circular boundary, some applications can be seen in the interaction of waves with arrays of circular cylinders [22], the free vibration of circular membranes [23] and circular plates [24] and flexural wave scattering [25] by using the addition theorem for Bessel functions. In the view point of mathematics, the procedure is exact and elegant. But we need to face the complicated formulation and the associated numerical calculation, and then its development is limited because the addition theorem involves complicated infinite series.

In this work, a collocation multipole approach is presented to semi-analytically solve the dynamic Green's function for an elastic infinitely extended isotropic solid (matrix) containing multiple circular inclusions with imperfect interfaces. The multiple expansions for the regular part of dynamic Green's functions are represented in terms of the Bessel or Hankel functions. Instead of using the complicated addition theorem, when considering the imperfect interface condition, the normal derivative with respect to non-local polar coordinate systems is exactly calculated by using the directional derivative. The imperfect interfaces condition can be satisfied by distributing collocation points along the interface of each inclusion. By truncating the high order terms of the multipole expansion, a coupled finite linear algebraic system is derived. The anti-plane displacement fields are obtained through the solution of the algebraic system. The proposed results are compared with available analytical solutions and numerical results using the FEM. Several numerical results are presented to investigate the influence of the frequency of excitation, imperfect interface and separation between inclusions on the dynamic Green's functions for the problem under consideration.

2. Problem statement and the general solution in the polar coordinate system

An elastic infinitely extended isotropic solid (matrix) containing N_I circular inclusions subjected to a time-harmonic anti-plane line force of strength $fe^{-i\omega t}$ located at ζ as shown in Fig. 1, where ω is the circular frequency, B_j denotes the jth boundary, $j=1,\ldots, N_I$. There are N_I+1 observer coordinate systems used to describe the present problem: (x, y) is a global Cartesian coordinate system centered at O; (r_j, θ_j) is the jth local polar coordinate system centered at O_j, attached to the center of the jth circle, with global Cartesian coordinates (x^j, y^j).

When a time-harmonic anti-plane line force is applied to the matrix, the induced out-of-plane displacement fields satisfy the following equation in the polar coordinate system

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_P(\boldsymbol{x})}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w_P(\boldsymbol{x})}{\partial\theta^2}+k_P^2 w_P(\boldsymbol{x})=-\frac{f}{\mu_P}\delta(\boldsymbol{x},\boldsymbol{\zeta}),\quad \boldsymbol{x}\in\Omega_P \tag{1}$$

where w_P is the anti-plane displacement field of the matrix, $\delta()$ is the Dirac delta function and $k_P=\omega/c_P$ is the wave number of the matrix. When the inclusion is subjected to a time-harmonic anti-plane line force, the governing equation for the anti-plane displacement field of the inclusion is given by the following equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_I(\boldsymbol{x})}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w_I(\boldsymbol{x})}{\partial\theta^2}+k_I^2 w_I(\boldsymbol{x})=-\frac{f}{\mu_I}\delta(\boldsymbol{x},\boldsymbol{\zeta}),\qquad \boldsymbol{x}\in\Omega_I \tag{2}$$

where $k_I=\omega/c_I$ is the wave number for the inclusion. μ_P and μ_I are the shear moduli of the matrix and inclusion, c_P and c_I are the shear wave speed, Ω_P and Ω_I are regions occupied by the matrix and inclusion, respectively. For the sake of convenience, the time factor $e^{-i\omega t}$ associated with all the field variables has been omitted. The induced stresses are defined by

$$\sigma_{zr}^P=\mu_P\frac{\partial w_P}{\partial r}\text{ and }\sigma_{z\theta}^P=\mu_P\frac{\partial w_P}{r\partial\theta},\qquad \boldsymbol{x}\in\Omega_P \tag{3}$$

$$\sigma_{zr}^I=\mu_I\frac{\partial w_I}{\partial r}\text{ and }\sigma_{z\theta}^I=\mu_I\frac{\partial w_I}{r\partial\theta},\qquad \boldsymbol{x}\in\Omega_I. \tag{4}$$

Moreover, the circular interface between the matrix and inclusion is assumed to be imperfect in this work. The imperfect interface boundary condition is given by Wang and Meguid [26]

$$\sigma_{zr}^I=\sigma_{zr}^P=\beta(w_P-w_I), \tag{5}$$

where the non-negative constant β is the imperfect interface parameter. The circular inclusion is perfectly bonded to the matrix as β approaches infinity. On the other hand, the circular inclusion is fully debonded to the matrix as β approaches zero. When the wave number k approaches zero, the Helmholtz equation is reduced to the Laplace equation.

In polar coordinates, the Helmholtz equation has separated solutions of the form

$$J_m(kr)e^{im\theta},\ Y_m(kr)e^{im\theta},\ H_m^{(1)}(kr)e^{im\theta}\text{ and }H_m^{(2)}(kr)e^{im\theta},$$

where J_m and Y_m are the mth-order Bessel functions of the first and the second kind, respectively, and $H_m^{(1)(2)}(kr)=J_m(kr)\pm iY_m(kr)$ are the Hankel functions of the first and the second kind [27]. Since the functional value of the Bessel function of Y_m is infinite at the origin, the permissible solution is

$$w^I(r,\theta)=a_0J_0(kr)+\sum_{m=1}^{\infty}(a_m\cos m\theta+b_m\sin m\theta)J_m(kr), \tag{6}$$

for the interior domain and

$$w^E(r,\theta)=a_0H_0^{(1)}(kr)+\sum_{m=1}^{\infty}(a_m\cos m\theta+b_m\sin m\theta)H_m^{(1)}(kr), \tag{7}$$

for the exterior domain, where the coefficients a_m and b_m are determined by using the imperfect interface boundary conditions.

3. The multipole method

The dynamic Green's function problem is to solve Eqs. (1) and (2) subjected to the imperfect interface conditions on each circular interface. From Eq (7), we can express the general solution of the