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Debonding of FRP and thin films from an elastic half-plane using a coupled FE-BIE model

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ABSTRACT

A Finite Element-Boundary Integral Equation (FE-BIE) coupling method is proposed to investigate a flexible bar weakly attached to an elastic orthotropic half-plane. Firstly, the analysis focused on the case of a bar subjected to horizontal forces and thermal loads considering interfacial displacements linearly proportional to the tangential traction. Secondly, the debonding behaviour of a composite reinforcement glued to a substrate has been modelled. Using an incremental nonlinear analysis, a bilinear elastic-softening interfacial traction-slip law has been implemented simulating the delamination of pure mode II. Finally, the influence of the anchorage length on the ultimate bearing capacity of the adhesive joint has been investigated.

1. Introduction

In the last few decades, strengthening of existing concrete and masonry structures [1], and rehabilitation of steel structures [2] have emerged as a cutting edge issue in structural engineering. Particularly, the use of fibre reinforced polymer (FRP) strips has become more and more common than ever before, as it has proved to be a rapid and efficient technical solution. Moreover, thin film-based devices and coated systems have been widely employed, remarkably in fields of aerospace and electronic engineering. There are plenty of studies focused on the issue of strengthening reinforced concrete (RC) members with externally bonded FRP sheets [3]. For these applications, a simple reference model may be a straight elastic stiffener of prescribed length bonded to an elastic substrate in plane state that can debond in pure mode II only. Moreover, bending stiffness of the stiffener may be disregarded because of negligible thickness. Consequently, the stiffener is not able to sustain transverse loads and no peeling stresses can arise at the interface.

In 1932, Melan studied the problem of a point force applied to an infinite stiffener bonded to an infinite linear elastic sheet [4]. Several authors have reconsidered and extended Melan's problem, especially for stiffened plate in aircraft structures and FRP strengthened RC structures. Early studies concerning stiffeners welded to an elastic substrate have adopted a series approximation method to solve singular integral equations including a proper Green function, see [5] and references cited therein. Perfect adherence hypothesis was relaxed in [6], where the adhesive interface was substituted by a set of independent linear elastic springs. This classical assumption [7] is frequently referred to as weak or imperfect interface and for a soft thin adhesive connecting two adherents was justified making use of asymptotic expansion methods of the corresponding three-dimensional elastic problem [8]. However, correction terms may be required at the adhesive ends [9]. For the case of an FRP plate glued to a rigid substrate, a closed-form analytical solution of shear-out test has been presented in [10], assuming an elastic-softening bilinear bond law at the adhesive interface and fracture behaviour in mode II along the interface. In the same framework, the effect of the substrate elasticity has been considered in [11,12], using a series approximation method. Alternatively, a stress analysis combined with linear elastic fracture mechanics can be used to evaluate the critical delamination condition for RC beams strengthened with FRP strips [13].

Finite element procedures based on continuum damage models are required whenever the fracture behaviour involves the substrate [14–16]. Accurate results have been obtained in [17–20] using a regularised extended FE approach to interpret delamination tests in FRP strengthened concrete. Nonetheless, the FE approach undergoes important limitations when applied to film-substrate systems [21,22] because a refined mesh has to be used to describe the thin layer of the film. Furthermore, to simulate the half-plane, FE meshes should be extended to a region significantly greater than the contact area; thus increasing the computational burden.

Boundary Element (BE) techniques can be used to evaluate the mechanical behaviour of coated systems involving thin layers, provided that the nearly-singular integrals arising in the BE formulations are correctly handled [23,24]. Symmetric Galerkin boundary element techniques for cohesive interface problems are presented in [25,26], where the nonlinear behaviour has been localised at the interface only. Moreover, reference [26] considered both substrate and reinforcement as lin-

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ear elastic bodies and showed that a bar model is computationally more efficient than that of a thin layer.

For bars and beams resting on two-dimensional substrates, a Finite Element-Boundary Integral Equation (FE-BIE) coupling method is well suited to provide very accurate solutions at a low computational cost. To date, several problems have been analysed with the FE-BIE coupling method, such as thin films bonded to an isotropic elastic substrate subjected to thermal or axial loads [27] and Euler-Bernoulli and Timoshenko beams in frictionless [29,30] or adhesive contact [31,32] with an elastic half-plane, including buckling problems [33,34].

In particular, the FE-BIE coupling method makes use of a mixed variational formulation including the Green function of the substrate, and assumes as independent fields both the nodal displacements and the contact tractions. It is worth noting that only the structure in contact with the substrate boundary has to be discretised. In addition, the mechanical response of the half-plane is represented through a weakly singular integral equation, whose solution is given analytically, avoiding singular and hyper-singular integrals typically involved in the classical BE formulation. For the mixed problem at hand, useful mathematical references are [35,36], where well-posedness of the variational problem and the corresponding Galerkin solution are set in the proper functional framework.

In the present paper, the FE-BIE coupling method is used introducing a slip between a flexible bar and an elastic orthotropic half-plane. To the authors' knowledge, the present proposal represents a new contribution.

First, the slip is assumed linearly proportional to the interface reactions. The cases of a bar subjected to a point force or a uniform thermal variation are investigated.

In the second part of this paper, incremental nonlinear analysis of the proposed model is adopted to investigate the delamination of an FRP strengthened RC substrate. The analysis of the interfacial reaction turns out to be important to predict the detachment phenomenon. The governing parameters of constitutive laws for adhesive interfaces must generally be estimated from experiments. However, the experimental determination of the mechanical properties of an adhesive is a complex task. These properties can be obtained by shear-out tests adopting different layouts, such as single slipping test with fixed back side or double pull-out shear schemes [37,38]. Simple formulations for debonding analysis are generally based on *a priori* analytical expressions describing the interface bond-slip law calibrated from experimental results. In these formulations, a fracture process in pure mode II is considered, disregarding the effects due to interface normal tractions (peeling) and out-of-plane displacements (uplift). The interface peeling stress and uplift develop due to eccentricity between applied force and interface and can be experimentally observed through advanced optical systems [39]. Although these components affect the ultimate bearing capacity of the adhesive joint, their influence on the distribution of interface slip throughout the contact region is negligible [40].

In the present model, an incremental analysis with displacement control has been used assuming a bilinear bond-slip law, and the results have been compared with those of experimental tests and analytical formulations found in the literature.

2. Variational formulation

An elastic bar with length L and cross section A attached to an elastic half-plane is considered, as shown in Fig. 1. Reference is made to a Cartesian coordinate system (O, x, z) centred at the midsection of the bar, with the vertical axis z directed toward the half-plane and the x-axis placed along the interface. Both the bar and the semi-infinite substrate are made of homogeneous and isotropic solids. Elastic constants E_b and ν_b respectively denote the Young modulus and the Poisson coefficient of the bar, whereas E_s and ν_s characterise the substrate. Generalised plane stress or plane strain regimes are considered. For plane strain, the width b of the half-plane will be assumed unitary. The thickness of the coating is assumed thin, so making possible to neglect its bending stiffness. In the absence of peeling stresses, only tangential tractions $r_x(x)$ occur along the contact region. The bar is subjected to a generically distributed horizontal load $p_x(x)$ or thermal variation $\Delta T(x)$.

Unlike the perfect adhesion case proposed in [27], the relaxed adhesion is representative of the mechanical characteristics of the adhesive connecting the bar with the substrate. This assumption involves the loss of continuity between bar displacement $u_{x,b}$ and half-plane displacement $u_{x,s}$.

2.1. Total potential energy for the bar

The strain energy of a bar can be written as follows [28]:

$$U_{\text{bar}} = \frac{1}{2}\int_L E_0 A(x)\left[u'_{x,b}(x) - \alpha_0 \Delta T\right]^2 dx, \tag{1}$$

where prime denotes differentiation with respect to x, and the Young modulus E_0 and the coefficient of thermal expansion α_0 of the bar are $E_0 = E_b$, $\alpha_0 = \alpha_b$ for a generalised plane stress, and $E_0 = E_b/(1-\nu_b^2)$, $\alpha_0 = (1+\nu_b)\alpha_b$ for a plane strain state. Noteworthy, the axial force in the bar is $N(x) = E_0 A(x)[u'_{x,b}(x) - \alpha_0 \Delta T]$. The potential energy Π_{bar} can be written as the strain energy U_{bar} minus the work related to the external loads:

$$\Pi_{\text{bar}} = U_{\text{bar}} - b\int_L \left[p_x(x) - r_x(x)\right] u_{x,b}(x)\, dx. \tag{2}$$

2.2. Total potential energy for the substrate

The solution to the elastic problem for a homogeneous isotropic half-plane loaded by a point force tangential to its boundary is referred to as the Cerruti solution [41]. For a point force $P_x(\hat{x})$ applied to the half-plane boundary at the coordinates $\hat{x}$ (Fig. 2), the closed form expression for the surface displacement $u_{x,s}(x) = g(x,\hat{x})P_x(\hat{x})$, where the Green function $g(x,\hat{x})$ is:

$$g(x,\hat{x}) = -\frac{2}{\pi E}\ln\frac{|x-\hat{x}|}{d}. \tag{3}$$

In Eq. (3), $E = E_s$ or $E = E_s/(1-\nu_s^2)$ in the plane stress or plane strain, respectively, and d is an arbitrary length associated with a rigid displacement. The horizontal displacement $u_{x,s}(x)$ due to the interfacial tractions $r_x(x)$ acting along the boundary between the half-plane and the bar can be found as

$$u_{x,s}(x) = \int_L g(x,\hat{x})\, r_x(\hat{x}) d\hat{x}. \tag{4}$$

Making use of the theorem of work and energy for exterior domains [42], it can be shown that the total potential energy Π_{soil} for the half-plane equals one half the work of external loads [27,29]:

$$\Pi_{\text{soil}} = -\frac{b}{2}\int_L r_x(x)\, u_{x,s}(x)\, dx. \tag{5}$$

By introducing Eq. (4) into Eq. (5), one obtains

$$\Pi_{\text{soil}} = -\frac{b}{2}\int_L r_x(x)\, dx \int_L g(x,\hat{x})\; r_x(\hat{x})\, d\hat{x}. \tag{6}$$

2.3. Total potential energy for the adhesive

A displacement jump occurs when a stiffener is glued to a support by means of an adhesive. In the following, the transmission traction r_x is assumed proportional to the slip $\Delta u_x = u_{x,b} - u_{x,s}$ between the bar and the half-plane displacements

$$r_x = k_x\, \Delta u_x, \tag{7}$$

where parameter k_x summarises the mechanical characteristics of the interface [8]. Making use of Eq. (7), the total potential energy for the adhesive can be written as

$$\Pi_{\text{spring}} = \frac{b}{2}\int_L r_x(x)\,\Delta u_x(x)\, dx - b\int_L r_x(x)\,\Delta u_x(x)\, dx = -\frac{b}{2}\int_L \frac{r_x^2(x)}{k_x}\, dx. \tag{8}$$