Engineering Analysis with Boundary Elements 94 (2018) 159–171

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Bending analysis of FG plates using a general third-order plate theory with modified couple stress effect and MLPG method

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ARTICLE INFO

Keywords:
Meshless local Petrov–Galerkin method
General third-order plate theory
Modified couple stress effect
Functionally graded plate
Moving least-square approximation

ABSTRACT

Meshless local Petrov–Galerkin analysis of functionally graded plates based on a general third-order shear deformation plate theory with a modified couple stress effect is presented. Governing equations of problem are a fourth-order partial differential equations system which derived in terms of eleven generalized displacement variable, by applying the principle of virtual displacements. The moving least-squares approach is used for approximation of unknown variables and the Gauss weight function is employed as test function for obtaining local weak form. The Gauss–Legendre quadrature method is utilized for numerical integration of weak equations. Static bending results of a simply-supported plate is obtained for various power law index and length scale parameter, and is compared to analytical solutions that shows high accuracy in results.

1. Introduction

In the mechanical problems, either time-dependent or time-independent, it is recommended to reduce three-dimensional governing equations of problems to a two-dimensional formulations by a proper method. One of the best approaches for reducing three-dimensional elasticity is representing a dimension of problem through power series and multiplying this series to the functions that describe roles of other two dimensions. For analyzing plate structures, it is preferred to define the direction of thickness by proper order of power series to account for the kinematic of deformation and derive constitutive relations. This is due to fact that thickness of plates is quite small compared to their in-plane dimensions. Types of two-dimensional plate theories consist of two categories, displacement-based and stress-based. These two categories are similar in expansion of the fields by increasing powers of the thickness coordinate, and different in their strain/stress compatibility conditions. Displacement-based theories are strain/stress compatible, therefore these theories have preferred in literature and in this research too. To find the optimal order of power series for expanding thickness coordinate in formulation, it should be considered that what degree of variation of strains and stresses is recommended to have better results. By expanding displacement field up to three order for in-plane displacements and two order for out-plane displacements in most of cases, the transverse shear strains have quadratic variation through the plate thickness. This approach is the third-order shear deformation theory (TSDT) and if all terms in power series remain in expansion, the theory is called the general third-order plate theory (GTPT). In addition to having proper variation for the transverse shear strains, TSDT or GTPT do not need to any shear correction factor unlike the first-order shear deformation theory (FSDT).

In conventional continuum mechanics, the effects of micro- and nano-scale interactions are not considered. For analyzing size-dependent behavior of micro-scale plate structures, the modified couple stress theory can be implemented in deriving governing equations. This theory has only one length scale parameter for representing microstructural effect. This is an advantage of the modified couple stress approach in comparison to the classical couple stress theories, due to complexity of determining two length scale parameter that each of them related together and to size-dependent effect.

A meshless method is a method used to establish system algebraic equations for the whole problem domain without the use of a predefined mesh for the domain discretization [1]. Meshless methods use a set of nodes scattered within the problem domain as well as sets of nodes scattered on the boundaries of the domain to represent (not discretize) the problem domain and its boundaries. These scattered nodes do not form a mesh, so it does not need to any a priori information on the relationship between scattered nodes for the interpolation or approximation of the unknown functions of field variables [2].

There are two basic forms of meshless methods: the global form and the local form. The meshless methods based on the global form can be applied easily for solving partial differential equations (PDE) and integral equations. But these methods are not advantageous for solv-

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https://doi.org/10.1016/j.enganabound.2018.06.015
Received 23 February 2018; Received in revised form 23 May 2018; Accepted 14 June 2018
0955-7997/

ing all categories of PDEs. To overcome these disadvantages, the meshless methods based on local approach have been introduced. These approaches consist of two classes [3]:

- Local meshless methods based on the variational weak form
- Local meshless methods based on the strong form.

In the first class, it is necessary to use a numerical integration procedure for solving the local weak equations. This procedure can be meshless or use background cells for discretizing integration domain.

In the numerous meshless methods that have applied for solving partial differential equations until today, the meshless local Petrov–Galerkin method (MLPG) is one of the few methods that is truly meshless, because this approach do not need to meshing either for the interpolation of unknown variables or for the numerical integration of weak equations. In other words, no domain or boundary element is required for discretizing the domain of problem. The MLPG method is based on local weak form that means for every node in the domain, weak form equations are satisfied in local sub-domains around nodes. In view of the fact that no global integration is involved, the MLPG method is addressed as a truly meshless method [4]. This method has been successfully applied for solving a wide range of problems in engineering [5].

1.1. Present study

After the MLPG method introduced by Atluri and Zhu [6] in 1998, and discussed in detail by Atluri and his co-authors [7–9], several researches is published until today which employed the MLPG approach in higher-order plate theories.

Qian et al. [10] analyzed deformation of a homogenous and isotropic thick plate with a higher-order shear and normal deformable plate theory (HOSNDPT) by using MLPG method that displacements according to thickness derived by assuming the Legendre polynomial as basis function. They also computed static deformations, and free and forced vibrations of a thick functionally graded plate by applying HOSNDPT and MLPG method [11]. Qian and Batra [12,13] studied transient thermoelastic deformations of a thick FG plate by HOSNDPT and MLPG approach. Comparison of the MLPG method and the finite element method (FEM) presented in this research. They also designed a FG plate for optimal natural frequencies by employing MLPG method and HOSNDPT [14]. Xiao et al. [15] analyzed thick FG plates by using HOSNDPT and MLPG approach. They expanded displacement field by the Legendre polynomial basis and utilized the Radial Basis Functions (RBFs) for interpolation of variables. Gilhooley et al. [16] and Xiao et al. [17] attempted this concept for analyzing thick FG plates and thick composite laminates, respectively.

In addition to MLPG method, several researches investigated solutions of higher-order plate theories by employing various methods and compared results with MLPG solutions. Ferreira et al. [18,19] solved TSDT governing equations of FG plates by using the collocation RBF method and compared their results with Qian et al. [11] solutions. Natural frequencies of a FG plate are achieved by employing the TSDT and the collocation RBF approach by Ferreira et al. [20] and compared with Qian et al. [11] results. Sheikholeslami and Saidi [21] obtained solutions of vibration of a FG plate with HOSNDPT and analytical approach, and compared their results with MLPG method [11].

Reddy and Kim [22] proposed the GTPT with the modified couple stress effect and derived governing equations by employing the principle of virtual displacements and the fundamental lemma of variational calculus. For first time, analytical solutions for bending, buckling and vibration of FG plates obtained by Kim and Reddy [23] by using Navier solution technique. They also computed solutions for bending of FG plates with von Kármán nonlinearity by implementing finite element method (FEM) with conforming element which has four degree of freedom per node [24].

In this paper, the GTPT governing equations with modified couple stress effect of FG plates are solved by using the MLPG method with the moving least-squares (MLS) approach for approximation of unknown variables.

2. Displacements and strains of GTPT

If in-plane displacement and out-of-plane displacements extend up to third power and second power of thickness direction, respectively, the displacement field of GTPT can be formulated as [25,26]:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\theta_x(x,y,t) + z^2\varphi_x(x,y,t) + z^3\psi_x(x,y,t) \\ u_2(x,y,z,t) &= v(x,y,t) + z\theta_y(x,y,t) + z^2\varphi_y(x,y,t) + z^3\psi_y(x,y,t) \\ u_3(x,y,z,t) &= w(x,y,t) + z\theta_z(x,y,t) + z^2\varphi_z(x,y,t) \end{aligned} \quad (1)$$

where u, v, w, θ_x, θ_y, θ_z, ϕ_x, ϕ_y, ϕ_z, ψ_x, and ψ_y are unknown generalized displacements. In this expansion, normality and straight condition of the transverse normal lines under the Kirchhoff assumptions are released because of cubic variation of in-plane displacements. Also, inextensibility of transverse normal lines is released due to quadratic variation of out-of-plane displacements. Here linearized strains are considered and their relations to the generalized displacements take the form:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \theta_z \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \theta_x + \frac{\partial w}{\partial x} \\ \theta_y + \frac{\partial w}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ 2\phi_z \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \\ 2\phi_x + \frac{\partial \theta_z}{\partial x} \\ 2\phi_y + \frac{\partial \theta_z}{\partial y} \end{Bmatrix} + z^2 \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \\ 3\psi_x + \frac{\partial \phi_z}{\partial x} \\ 3\psi_y + \frac{\partial \phi_z}{\partial y} \end{Bmatrix} + z^3 \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ 0 \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \\ 0 \\ 0 \end{Bmatrix} \quad (2)$$

3. Governing equations of GTPT

For deriving governing equations of GTPT based on the modified couple stress theory of FG plates, it is needed to represent modified couple stress model and constitutive relations of considered FG plate.

3.1. Modified couple stress model

Yang et al. [27] proposed a modification of the classical couple stress theory (see [28]), which established that the couple stress tensor is symmetric and the symmetric curvature tensor is the only proper conjugate strain criterion that enters in equation of the total strain energy of the body. In the modified couple stress theory, strain energy density function depends only on the strain and the symmetric part of the curvature tensor, therefore only one length scale parameter is involved. This fact and inclusion of a symmetric couple stress tensor are two main advantages of the modified couple stress model over the classical couple stress theory. Virtual strain energy $\delta\mathcal{U}$ using modified couple stress model is defined as [29]:

$$\delta\mathcal{U} = \int_V (\delta\boldsymbol{\varepsilon} : \boldsymbol{\sigma} + \delta\boldsymbol{\chi} : \mathbf{m})\, dv \quad (3)$$

where summation on repeated indices is implied: here σ_{ij} are Cartesian components of the symmetric part of the stress tensor, ε_{ij} denotes the strain components, m_{ij} are the components of the deviatoric part of the symmetric couple stress tensor, and χ_{ij} donates the symmetric curvature tensor components and can be written as [30]:

$$\boldsymbol{\chi} = \frac{1}{2}\left[\nabla\boldsymbol{\omega} + (\nabla\boldsymbol{\omega})^T\right], \quad \boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{u} \quad (4)$$

or

$$\chi_{ij} = \frac{1}{2}\left(\frac{\partial \omega_i}{\partial x_j} + \frac{\partial \omega_j}{\partial x_i}\right), \quad i,j = 1,2,3 \quad (5)$$