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Analysis of surface cracks in round bars using dual boundary element method

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ABSTRACT

A closed-form solution for stress intensity factors (SIFs) of a semi-elliptical surface crack in round bars subjected to torsional loading using the dual boundary element method are presented in this paper. The SIFs of surface cracks having different crack aspect ratios and inclination angles were calculated using the numerical software BEASY. In general, the results revealed that the Mode III SIF was strongly affected by both the crack size and crack shape whereas that crack aspect ratio had the greatest influence on the SIFs regardless of the fracture modes.

1. Introduction

Cylindrical bars are used extensively in many machine applications either for transmitting power between parallel planes such as in automotive power trains, rotors or electrical machinery and to support combine cyclic torsion moment-tension loading during service. Often, these bars are subjected to fatigue stresses which may shortened its life span during service. Failure of such bars is frequently being associated with the presences of surface flaws. In particular, surface cracks may be present either due to metallurgical defects, improper surface treatment during manufacturing or initiated at notches, pinholes, etc. Hence, in order to mitigate failure of cylindrical bars associated with surface cracks requires the understanding of crack behavior when subjected to torsion moments [1]. A crucial part in the development of fatigue life prediction model is quantifying the severity of a surface flaws or cracks. In order to evaluate the crack behavior, linear elastic fracture mechanics (LEFM) approach has been the preferred choice. This approach requires the evaluation of the stress intensity factor (SIF) in the vicinity of a crack front [2].

Many researchers have reported on the SIF solutions for various surface cracks in cylindrical bars. Cai et al. [3] studied the threshold of crack propagation and derived a unified formula to estimate threshold of crack growth with respect to mean SIF from the view of fatigue limit condition at crack front. Mahbadi [4] used an approximation method to obtain equations for stress intensity factor of functional graded solid cylinders having radial crack located at inside or edge of the cylinder. The SIFs were obtained for both of plane stress and plane strain conditions of a rotating cylinder subjected to thermomechanical loading. By using the distributed dislocation technique in the analysis of an orthotropic circular cross section bar weakened by multiple cracks, Hassani and Monfared [5] concluded that the stress intensity factor of the crack tips and torsional rigidity were depended on critical factors such as the distance of the crack tip from the free boundary of the cross section, orthotropic ratio, crack length and interaction between the cracks. Zareei and Nabavi [6] employed a three-dimensional finite element method to calculate the SIFs for semi-elliptical cracks in pipes subjected to any arbitrary load. A finite element alternating method was employed by Kamaya and Nishioka [7] to deduce the SIFs of surface cracks in a cylinder. He and Hutchison [8] modeled a semi-elliptical surface crack in the infinite body with aspect ratios of $a/c = 0.5$ and 1 in ABAQUS and used a general purpose finite element code for the analysis. Many numerical methods using finite element method have been developed to obtain SIF values of surface cracks [9–18]

The dual boundary element method (DBEM) have also been adopted by many researchers to study the shape sensitivity analysis of SIFs with respect to the crack geometry [19–27], in the analysis of reinforced cracked plates [25–29] and in the analysis of three-dimensional mixed-mode crack growth [19,20,30]. For example, Siow et al. [23] studied SIFs of a corner crack in metals subjected to multiaxial fatigue loading. In this work, the authors used DBEM to evaluate SIFs in prismatic bar under cyclic bending and torsional loading. Similarly, Purbolaksono et al. [24] used DBEM to study SIFs of multiple semi-elliptical cracks in bi-material tubes under internal pressure. DBEM was particularly found to be suitable for quantifying the conservation integrals. In contrast to

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Nomenclature

a	depth of crack
c	half-length of crack
c'	half-length of distance between intersections of crack front with round bar surface
τ_0	applied torsion to the round bar
d	diameter of the round bar
r	radius of the round bar
L	length of the round bar
θ	crack face inclination
E	modulus of elasticity
ν	Poisson's ratio
s	crack front arc-length
h	element size
ϕ	parametric angle of ellipse
Q	shape factor for elliptical crack
K_I	mode I, SIF
K_{II}	mode II, SIF
δK_{II}	correction to Mode II, SIF
K_{III}	mode II, SIF
δK_{III}	correction to Mode III, SIF
k	elliptical modulus
$K(k)$	incomplete elliptic integral of the first kind
$E(k)$	incomplete elliptic integral of the second kind
F_{RM}	normalized SIFs for surface crack in the round bar
f_S	normalized value for SIFs at $\phi = \pi/2$ for Mode I & III and $\phi = \phi_0$ for Mode II
f_θ	inclination-correction factor
g	curve fitting function
CPE	corner points on the ellipse
DPE	deepest point on ellipse

FEM, the high accuracy of the displacements and stresses including their derivatives at the internal points can be reached by using DBEM through their boundary integral representations for the simulation of the crack growth process, especially for three-dimensional analysis [25,21,31]. Besides using FEM or DBEM, other studies have also proposed a closed-form solution by utilizing the weight function method for calculating the SIFs of a surface crack [32–36].

In general, the body of literatures indicated that a unanimous agreement of an acceptable closed-form solution for determining the SIFs of a surface crack has yet to be attained, especially for round bars subjected to torsion moment. In this research, the closed-form solutions of the SIFs for a surface crack in a solid round bar subjected to torsion moment was investigated based on the dual boundary element method (DBEM) using the computer program BEASY [37]. In addition, the results obtained via BEASY were compared and validated by using the proposed solutions of He and Hutchinson [8] for similar crack geometries.

2. Model and methodology

2.1. Model

The three-dimensional analysis is performed by using DBEM based software BEASY to evaluate SIFs of semi-elliptical surface cracks in the round bar subjected to pure torsion. Main parameters which being considered in this study are crack size ratio (a/d), crack aspect ratio (a/c) and crack inclination angle (θ) (in the direction where loading tends to open the crack face) and distinct sets of simulations have been executed to study influences of each factor on SIFs, properly. A round bar with diameter of $d = 10$mm and length of $L = 40$mm is used in this study (except for that set of simulations performed to evaluate crack size ratio in which round bar has different dimensions). Crack depth defines by a and its half-length by c at the surface where it is perpendicular to the a. Schematic of the surface crack modelled in BEASY is presented in Fig. 1. The solid round bar considered in this analysis was a 7000 series aluminum alloy having a yield stress of 500 MPa, modulus of elasticity of 70 GPa and Poisson's ratio ν of 0.33. In the present study, the maximum shear stress of $\tau_0 =$ 100 MPa is applied at the outer surface of the round bar as depicted in Fig. 1 and is within the elastic limit of the round bar.

It is evident that parametric angle ϕ for cracks in infinite solid body start from zero and goes to π but it can be seen in Fig. 1 that in the round bar parametric angle does not have the same range due to the bar's curvature. In the round bar parametric angle is $\phi_0 \le \phi \le \pi - \phi_0$ and ϕ_0 can be obtained by

$$\varphi_0 = \tan^{-1}\left(\frac{a}{\sqrt{4r^2 - a^2}}\right) \qquad \text{if} \quad a = c \text{ and } \theta = 0 \tag{1}$$

Else

$$\varphi_0 = \left|\tan^{-1}\left(\frac{\cos(\theta)\left(\sqrt{a^2 r_\theta{}^4 + c^4 r^2 - a^2 c^2 r_\theta{}^2} - a r_\theta{}^2\right)}{\sqrt{a\left(2r^2\sqrt{a^2 r_\theta{}^4 + c^4 r^2 - a^2 c^2 r_\theta{}^2} + a^3 r_\theta{}^2 - a c^2 r^2 - 2 a r^2 r_\theta{}^2\right)}}\right)\right| \tag{2}$$

$$r_\theta = \frac{r}{\cos\theta} \tag{3}$$

$$r = \frac{d}{2} \tag{4}$$

However, in practical cases like existing cracks on the components it is impossible to define parameter c. According to Fig. 2, it is easier to measure a and c'. In this case Eqs. (7) and (8) should be implemented to calculate ϕ_0 and c, respectively.

$$\beta = \cos^{-1}\left(\frac{c'}{r_\theta}\right) \tag{5}$$

$$a' = r(1 - \sin\beta) \tag{6}$$

$$\varphi_0 = \sin^{-1}\left(\frac{a'}{a}\right) \tag{7}$$

$$c = \frac{c'}{\cos\varphi_0} \tag{8}$$

2.2. Application of DBEM in BEASY

Dual boundary element method (DBEM) which was developed by Mi and Aliabadi [20] is implemented in BEASY software to treat the crack boundaries. This method is based on the displacement and the traction integral equations that can be written, respectively, as [20]:

$$C_{ij}(\mathbf{x}')u_j(\mathbf{x}') = \int_S U_{ij}(\mathbf{x}',\mathbf{x})t_j(\mathbf{x})dS - \int_S T_{ij}(\mathbf{x}',\mathbf{x})u_j(\mathbf{x})dS \tag{9}$$

$$\frac{1}{2}t_j(\mathbf{x}') + n_i(\mathbf{x}')\int_S S_{kij}(\mathbf{x}',\mathbf{x})u_k(\mathbf{x})dS = n_i(\mathbf{x}')\int_S D_{kij}(\mathbf{x}',\mathbf{x})t_k(\mathbf{x})d\Gamma(\mathbf{x}) \tag{10}$$

Where x and x′ are boundary and source point, respectively. i and j represent Cartesian components, the coefficient C_{ij} is given by δ_{ii} (Kronecker delta function) for a smooth boundary at x′, $T_{ij}(x',x)$ and $U_{ij}(x',x)$ denote the Kelvin traction and displacement fundamental solutions, respectively, S_{kij} and D_{kij} contains derivatives of $T_{ij}(x',x)$ and $U_{ij}(x',x)$, n_i is the outward normal vector and S represents the domain surface.

These fundamental integrals in Eqs. (9) and (10) are regularly provided for $R \neq 0 (R = |x - x'|)$. Since these solutions are order of $1/R$ in U_{ij}, $1/R^2$ in T_{ij} and D_{kij} and $1/R^3$ in S_{kij}, they will exhibit singularities