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Homogenization technique for heterogeneous composite materials using meshless methods

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A B S T R A C T

Due to its random fibre distribution across the cross-section and their anisotropic and heterogeneous characteristic, the prediction of the mechanical behaviour of fibre composite materials is complex. Multi-scale approaches have been proposed in the literature to more accurately predict their mechanical properties using computational homogenization procedures.

This work is based on existing multi-scale numerical transition techniques suitable for simulating heterogeneous materials and makes use of two meshless methods—the Radial Point Interpolation Method (RPIM) and the Natural Neighbour Radial Point Interpolation Method (NNRPIM)—and the Finite Element Method (FEM). Representative volume elements (RVEs) are modelled and discretized using the three numerical methods. Prescribed microscopic displacements are imposed on different RVEs whose boundaries are periodic and, from the obtained stress field, the average stresses are determined. Consequentially, the effective elastic properties of a heterogeneous material are obtained for different fibre volume fractions. In the end, the numerical solutions are compared with the solutions proposed in the literature and it is proved that the NNRPIM achieve more accurate solutions than the RPIM and the FEM.

1. Introduction

Presently, fibre composite materials are considerably used in several industries such as aircraft, aerospace or automotive, due to their high specific strength and low weight, high corrosion and chemical resistance and reduced cost. Since they are widely used in primary structural components, composite materials have been considerably investigated in order to predict their failure mechanisms (such as delamination, micro-buckling of fibres, cross cracking, etc.). Consequently, it is vital to accurately predict their mechanical behaviour, which is a complex task due to their anisotropic and heterogeneous characteristic as well as their random fibre distribution across the cross-section. [1].

When analysing a composite material, two approaches can be identified: the macromechanical analysis and the micromechanical analysis. In the macromechanical analysis are not considered the micro-heterogeneities of the composite material and, therefore, it is treated as a homogeneous orthotropic continuum [1]. Despite this approach can be reliable in several applications, for more complex situations (such as micro-cracking), the micromechanical analysis is the most appropriate approach since the microscopic phenomena have great influence on the macroscopic behaviour of the material [2]. Thus, a multi-scale approach needs to be defined concerning the two scales: macro and micro. For simplicity reasons, most microscale models assume a periodic arrangement of the fibres in a fibre composite material and it is used a representative volume element (RVE), which statistically represents the microstructure of the material and contains the information of the elastic constants and fibre volume fraction of the composite material. The RVE can be considered as an infinitesimal point of the macroscale. Thus, using the scale transition theory, it is possible to identify the homogenized elastic properties at any infinitesimal point of the composite structure and, consequentially, have a more reliable constitutive model.

This project aims to develop a new computational tool—using MATLAB®—capable of being applied to a wide range of heterogeneous materials whose macro-scale behaviour cannot be interpreted or predicted without considering the complex processes that occur in lower dimensional scales. This research is based on existent multi-scale numerical transition techniques [3–8] suitable for simulating heterogeneous materials. Thus, this work intends to develop and to implement new numerical techniques using advanced discretization methods—meshless methods. The novelty of the present research relies on the use of a recently

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developed meshless method—the Natural Neighbour Radial Point Interpolation Method (NNRPIM)—in the framework of a material homogenization procedure. In the end, the advantages of using this meshless method in micromechanics are explained using comparison studies.

2. Meshless formulations

By opposition to the traditional finite element method (FEM), in the meshless methods, the concept of mesh or element is inexistent. In these methods, the nodes can be arbitrary distributed and the field functions are approximated within an 'influence-domain' rather than an element [9]. The 'influence-domain' is an area or volume (depending if the studied phenomenon is a 2D or a 3D problem) concentric with an interesting point or an 'influence-cell' that is constructed in the problem domain resulting in a node dependent integration background mesh [9]. As consequence, meshless methods could be divided into two categories, depending on how the numerical integration is done: the 'truly' meshless methods and 'not truly' meshless methods. The truly meshless methods are capable to construct background integration meshes based exclusively on the nodal cloud discretizing the problem domain. The other non-truly meshless approaches, use a background integration mesh constructed based on regular or irregular integration lattice following the Gauss–Legendre quadrature scheme, eliminating the mesh-free characteristic of these methods [9].

Also in opposition to the FEM (which has a no-overlap rule between elements), in meshless methods the nodal connectivity is imposed by the overlap of the 'influence-domains' [9]. The high nodal connectivity, and the fact that they are not mesh reliant makes meshless methods advanced discretization techniques that are solid alternatives to the FEM, especially in problems involving large deformations or fracture mechanics—which frequently are associated, in the FEM, with re-meshing procedures showing high computational costs.

In meshless methods, the shape functions have virtually a higher order, allowing a higher continuity and reproducibility [9]. Meshless methods can easily handle situations where the geometry is transitory, such as the aforementioned cases of large deformations or crack propagation problems. Within meshless methods, the nodal discretization can be easily changed (by adding or removing nodes), simplifying the refinement procedure [10]. In addition, the solutions obtained from the meshless methods can be more accurate when compared with a lower order FEM [10].

Although the existence of meshless methods is dated from 1977, with the introduction of the Smooth Particle Hydrodynamics Method (SPH) [11], the first global weak form based meshless method was only presented in 1994 with the Element Free Galerkin Method (EFGM) [12]. Besides the EFGM and the SPH, other very popular meshless methods are: the Meshless Local Petrov–Galerkin Method (MLPG) [13], the Reproducing Kernel Particle Method (RKPM) [14], Point Interpolation Method (PIM) [15], the Point Assembly Method [16], the Radial Point Interpolation Method (RPIM) [17] or the Natural Neighbour Radial Point Interpolation Method (NNRPIM) [9,18].

As already mentioned, this work combines a classical homogenization technique for multiscale modelling with two radial point interpolation meshless methods—the RPIM and the NNRPIM. The RPIM main advantage is its potential to be directly included or combined with FEM codes. Recall that in order to discretize the problem domain, the FEM discretizes the solid volume in a mesh of elements and its respective nodes. The RPIM can use the element's nodal mesh as the discretization nodal mesh and the elements as the background integration cells (in which integrations points are distributed). Additionally, the literature shows that for the same nodal mesh, the RPIM is capable to obtain more accurate solutions when compared with the FEM [9]. Thus, with the RPIM it is expected to achieve more accurate homogenized material properties, enhancing the precision of the existent multiscale modelling techniques and FEM computational frameworks.

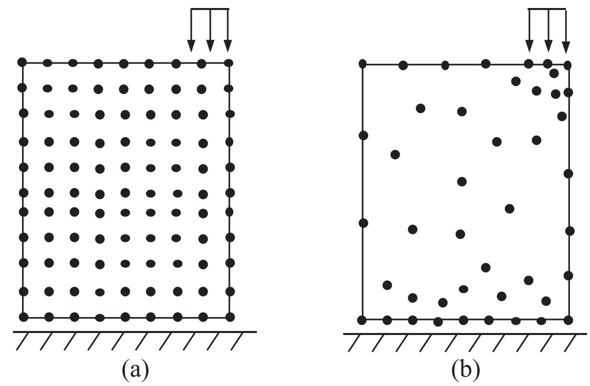

Fig. 1. (a) Regular nodal mesh. (b) Irregular nodal mesh.

Concerning the NNRPIM, this meshless method has the advantage of only requiring a nodal set to fully discretize the problem domain. This discretization flexibility opens new technological possibilities in multiscale modelling. For instances, using a micro-CT scan of a composite material it would be possible to discretize directly the problem domain (to each pixel it would correspond a node, for example). In a micro-CT scan, to the pixel's spatial information is associated a grey-intensity value. Therefore, the composite matrix and the fibre will show distinct grey-intensity values, allowing to identify and discretize directly several realistic RVEs with a nodal grid (corresponding to the pixels) and associating to each node a material type (from the pixel grey-intensity value). Additionally, combined with the NNRPIM higher accuracy [9], it is expected to obtain more reliable numerical results, i.e., more reliable materials properties.

2.1. *Meshless generic procedure*

Most of the meshless methods, such as the RPIM and the NNRPIM follow a standard procedure. After the description of the problem domain (with the essential and natural boundary conditions), its volume is discretized in a nodal mesh (the nodal discretization can be regular—Fig. 1(a)—or irregular—Fig. 1(b)—with the last one having, in general, a lower accuracy). However, in some problems where the locations of the stress concentration are predictable (crack propagation, holes, clamped boundaries, etc.), it is necessary to have a higher nodal density in those zones, which will lead to better results. Thus, it is essential to choose a correct nodal density of the mesh and the best nodal distribution possible without conducting to a significant increase in the computational cost, since these discretization parameters influence the method performance. An unbalanced distribution of the nodes could lead to less accurate results [9].

After the nodal discretization, a background integration mesh is constructed in the RPIM and in the NNRPIM, which is used to integrate the differential equations of the Galerkin weak form (these two meshless methods uses the same weak form formulation—Appendix A—as the FEM). The integration mesh can be nodal dependent (NNRPIM) or nodal independent (RPIM) [9]. A nodal independent integration mesh, in general, uses Gauss points, as in the FEM, fitted to the problem domain (Fig. 2(a)) or not (eliminating the Gauss points that are outside the problem domain—Fig. 2(b)). Another way to integrate the weak form equations is using the nodal integration, which can be achieved for instance by means of the concept of natural neighbours and the Voronoï diagrams. Here, the nodal mesh is used to construct the integration mesh, Fig. 2(c). The NNRPIM uses an improved version of this last mentioned technique.

After the definition of the integration mesh, the nodal connectivity can be imposed. While in the FEM this nodal connectivity is enforced with the interaction of the finite elements with the neighbour elements (there is a no-overlap rule between elements) and also at the element level (where the nodes belonging to the same element interact with