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DBEM and DRBEM solutions to 2D transient convection-diffusion-reaction type equations

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ABSTRACT

The present study focuses on the numerical solution of the transient convection-diffusion-reaction equation by transforming it into modified Helmholtz equation through an exponential type transformation. In the spatial discretization of the problem domain two different boundary element methods (BEM), namely the domain BEM (DBEM) and the dual reciprocity BEM (DRBEM), are employed which are combined with an implicit backward finite difference time integration. The BEM techniques differ in the sense of treating the leftover domain integral. That is, in DBEM the domain integral is kept and calculated by quadrature while it is reduced to an equivalent boundary interal by means of radial basis functions in DRBEM. The numerical simulations are first carried out for several values of diffusion coefficient in the transient convection-diffusion-reaction equation. The results reveal that DBEM gives more accurate results for smaller values of diffusion coefficient compared to DRBEM. Thus, DBEM is further used for the solution of the coupled transient convection-diffusion type magnetohydrodynamic flow equations. The results are presented by equivelocity and current lines for various values of problem parameters, which show the well-known characteristics of the MHD flow.

1. Introduction

The solution of convection-diffusion-reaction (CDR) equation has attracted attention of many researchers due to its various applications in biology, ecology, engineering and medicine. These type of equations represent quantities such as population size or concentration of nutrients, pollutants and other chemicals in the atmosphere, groundwater and surface water. For example, the model which describes the self-purification of a river is formulated in terms of the biological oxygen demand and the dissolved oxygen concentration by using convection-diffusion-reaction equations [1]. Moreover, the analysis and the computation of solution of the CDR equations from mathematical biology have been important for the understanding of biological processes, to verify the hypotheses about the underlying biology and to apply these kind of models to patient's specific data in medicine. The models concerning the tumour invasion, tumour angiogenesis and bacterial pattern formation are also described by convection-diffusion-reaction equations [2]. These models are solved, in general, using numerical techniques. Among these, the domain discretization techniques such as finite difference method (FDM) and finite element method (FEM) have been widely used for the solution of both steady and transient convection or diffusion dominated problems. There have been many studies on the steady case, however, the works carried only on the transient problems will be mentioned here, since this study focuses on the solution of transient equations. Clavero and Gracia [3] have solved the transient convection-diffusion problem by using FDM while a combined FDM and FEM has been employed by Douglas and Russell [4]. On the other hand, FEM has been widely used for the discretization of time dependent convection-diffusion-reaction equations by Tezduyar et al. [5], John and Schmeyer [6] with small diffusion parameter, Burman and Fernández [7] with symmetric stabilization, and Codina and Blasco [8] with subgrid scales. Furthermore, a discontinuous Galerkin FEM has been proposed for the space-time discretization of a CDR equation in the work [9]. In [10], a comparison of some FEM approaches, such as streamline-upwind Petrov–Galerkin and least-squares, has been presented to solve the CDR-problems. On the other hand, the boundary element method (BEM) which is an efficient alternative to domain discretization techniques due to its boundary-only nature, has been recently employed for the solution of convection-diffusion type equations in the works [11–13]. The transient advection-diffusion problem has been studied by Cunha et al. [14] using both DBEM and time domain BEM (TDBEM), and by Singh and Tanaka [15] using DRBEM. Moreover, an Eulerian–Lagrangian BEM technique has been proposed in the work [16].

Another application area that one can face with the convection-diffusion type equations is the magnetohydrodynamics (MHD) which studies the flow resulting from the interaction between the magnetic field and the moving electrically conducting fluids. The governing equations of MHD flow coupled in the velocity and the induced magnetic field are derived from the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism through Ohm's law

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[17–19]. The MHD flow problem in channels has attracted attention due to its wide range of engineering applications such as power generation, acceleration, geothermal energy extraction, conducting plasma in physics, producting liquid metals, nuclear fusion, etc. The early FEM formulations for MHD flow are given in 2D by Singh and Lal in [20] and in 3D by Salah [21], while the FDM has been applied in [22,23]. Recently, a meshless local boundary integral equation method, a meshless local Petrov-Galerkin method and a localized meshless point collocation method are employed in the works [24,25] and Loukopoulos [26], respectively, to solve the unsteady MHD problems. Moreover Bozkaya and Tezer-Sezgin [27] have presented a combination of DRBEM with differential quadrature method for spatial and time discretization, respectively, for solving the MHD flow in rectangular ducts. Finally, a direct BEM formulation with the time-dependent fundamental solution has been proposed by Bozkaya and Tezer-Sezgin [28].

The aim of this paper is to solve the transient convection-diffusion-reaction and MHD duct flow equations by using the DRBEM and DBEM with the fundamental solution of modified Helmholtz equation. Therefore, the physical problems modelled in terms of a transient convection-diffusion-reaction equation are transformed first into a modified Helmholtz equation by the use of a time-dependent exponential type transformation. The code validation for both DBEM and DRBEM is carried out by solving a transient convection-diffusion-reaction problem which has an analytical solution. Then, the DBEM is employed for the first time to the best of authors' knowledge in order to solve the transient MHD duct flow with both insulated and variable conductivity walls. The MHD equations are in the coupled convection-diffusion type equations which can be also transformed into two decoupled modified Helmholtz equations using the same exponential type transformation including time levels. This enables one to use the fundamental solution of modified Helmholtz equation through the application of DBEM. The velocity and the induced magnetic field are simulated for increasing values of Hartmann number and several wall conductivity parameters. The DBEM captures the well-known behavior of the MHD duct flow very well.

2. Problem definition and numerical methods

The general time-dependent convection-diffusion-reaction equation is given by

$$\frac{\partial u}{\partial t} - \varepsilon\nabla^2 u + \vec{a}.\nabla u + \sigma u = h \quad \text{in} \quad (0,T]\times\Omega \tag{1}$$

where $u(x, t)$ is the solution, ε and $\vec{a}=(a_1,a_2)$ denote the diffusion and convection coefficients, respectively, σ is the reaction coefficient and h is a potential source term in a considered finite time interval T. The mixed type boundary and the initial conditions are given as

$$\alpha u + \mu\frac{\partial u}{\partial n} = g \quad \text{on} \quad [0,T]\times\Gamma, \qquad u(x,0) = y_0 \quad \text{where} \quad x\in\Omega. \tag{2}$$

Here, α and μ are constants, $g = g(x,y,t)$ is a given function and Γ is the boundary of the spatial domain Ω.

In the present study, the focus is on the numerical solution of Eq. (1) by DRBEM and DBEM which make use of the fundamental solution of modified Helmholtz equation. Therefore, the CDR Eq. (1) with constant convection coefficient $\vec{a}=(a_1,a_2)$ can be transformed into the modified Helmholtz equation by using a time dependent exponential type transformation [14]

$$u(x,y,t) = \exp\left(\frac{a_1r_x + a_2r_y}{2\varepsilon}\right)\phi(x,y,t). \tag{3}$$

Thus, Eq. (1) reduces to the following inhomogeneous modified Helmholtz equation

$$\nabla^2\phi - s^2\phi = \frac{1}{\varepsilon}\left(\frac{\partial\phi}{\partial t} - h_1\right) \tag{4}$$

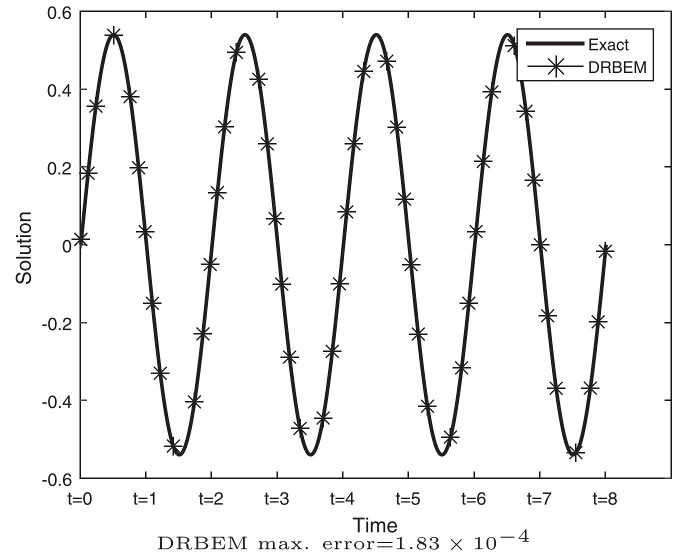

Fig. 1. Time evolution of the exact and DRBEM solutions at (0.5, 0.5), $T = 8$, $\varepsilon = 1$.

where

$$h_1 = h\exp\left(-\frac{a_1r_x + a_2r_y}{2\varepsilon}\right), \qquad s = \sqrt{\frac{\sigma}{\varepsilon} + \frac{a_1^2 + a_2^2}{4\varepsilon^2}}. \tag{5}$$

Here, r is the magnitude of the position vector $\vec{r}=(r_x,r_y)$ between the source point $\xi=(x_i,y_i)$ and field point (x, y). The spatial discretization of the modifed Helmholtz Eq. (4) is performed by using two BEM techniques, namely DRBEM and DBEM, with the fundamental solution of the modified Helmholtz equation $u^* = \frac{1}{2\pi}K_0(sr)$. Here, $K_0(sr)$ is the modified Bessel function of the second kind and of order zero. The aim of the boundary element method is to transform the given differential equations into equivalent integral equations through inherent use of the fundamental solutions for the whole governing equations. Such fundamental solutions are not generally available. Thus, some alternative techniques, namely DRBEM and DBEM, in which the fundamental solutions to only a piece of the governing equations are used, have been developed. The application of alternative BEM techniques results in an integral equation with a leftover domain integral. In DRBEM, the leftover domain integral is transformed into an equivalent boundary integral by means of radial basis functions while it is preserved and evaluated by the use of numerical integration in DBEM.

2.1. DRBEM formulation

In DRBEM, by weighting Eq. (4) with the fundamental solution of modified Helmholtz equation u^* and applying the Green's second identity, one can obtain the following integral equation [29]:

$$c_i\phi_i + \int_\Gamma q^*\phi d\Gamma - \int_\Gamma u^*\frac{\partial\phi}{\partial n}d\Gamma = -\int_\Omega \frac{1}{\varepsilon}\left(\frac{\partial\phi}{\partial t} - h_1\right)u^* d\Omega \tag{6}$$

where subscript i denotes the source point $\xi=(x_i,y_i)$. The coefficient c_i is given as $c_i = \theta_i/2\pi$ with θ_i is the internal angle at the source point i and $q^* = \frac{\partial u^*}{\partial n}$. The domain integral on the right hand side of Eq. (6) is approximated by using radial basis functions f_j as

$$\frac{1}{\varepsilon}\left(\frac{\partial\phi}{\partial t} - h_1\right) \approx \sum_{j=1}^{N+L}\alpha_j(t)f_j(x,y) \tag{7}$$

in which the functions f_j are linked to the particular solutions $\hat{u}_j$ of the nonhomogeneous modified Helmholtz equation $\nabla^2\hat{u}_j - s^2\hat{u}_j = f_j$. Sub-

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