Engineering Analysis with Boundary Elements 91 (2018) 1–6

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Computation of electric field inside substations with boundary element methods and adaptive cross approximation

Ana Drandić, Bojan Trkulja*

Department of Fundamentals of Electrical Engineering and Measurements, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia

ARTICLE INFO

Keywords:
Boundary element methods
Electric field
Adaptive cross approximation
Integral equations
Substations

ABSTRACT

Power system substations and HV power lines are sources of electric and magnetic fields of industrial frequency. Guidelines for permissible electric and magnetic field strengths regarding human exposure and electromagnetic compatibility (EMC) are provided by standards. In this paper a novel method for calculating 3D electric field near power system facilities is presented. Computation procedure is based on integral equations and boundary element methods. The influence of polynomial order of functions used to approximate the unknown linear charge densities over the elements is tested for constant, linear and cubic spline functions. Due to a complex geometry of substations, a large system of equations needs to be solved to find the unknown charge densities over the elements of the model. In order to reduce computational efforts adaptive cross approximation (ACA) is employed. The results of computation are in a good agreement with measurements.

1. Introduction

In last decades the consumption of electricity is continuously growing. Electrical power transmission to consumers is carried out using high voltage transmission lines and substations. Transmission lines and substations are sources of electromagnetic fields. Voltage levels of the power transmission are increasing and there is a high penetration on the market of additional artificial sources used in communication and networking.

General public is concerned about possible health risk due to exposure to electric and magnetic fields. There are several studies indicating biological responses from power frequency electromagnetic fields [1–3]. The International Commission on Non-Ionizing Radiation Protection (ICNIRP) provided guidelines regarding general public exposure and occupational exposure [4].

Due to the fact that the power frequency is 50 Hz or 60 Hz, time-harmonic electric and magnetic fields are considered quasistatic [5]. Therefore, electric field is calculated separately.

Different approaches for calculation of electric field of transmission lines and substations exist. Simple geometries can be calculated with 2D models of the problem [6]. Complex geometries of substations need 3D modeling. *Charge simulation methods* (CSM) can be implemented in computation of electric field inside substations [7]. Hybrid of CSM and *boundary element methods* (BEM) is presented in [8] to model influence of apparatus constructions. Parallelized BEM with cubic spline approximation over linear elements and bicubic splines approximation over conductive surfaces of apparatus constructions presented in [9] shows accurate computation inside substations.

Significant efforts are conducted in measurements of electromagnetic fields inside and in vicinity of outdoor substations. Measurement of power frequency electric and magnetic fields inside substations and under transmission lines is performed in different countries [10]. Measured electric and magnetic fields in substations are compared to guidelines [11]. Analysis of environmental influence of UHV transmission lines and substations are subject of particular interest [12,13]. In [15] computation of electric fields inside a substation is done with *finite element methods* (FEM) and compared with measurements.

In this paper electric field inside substation is calculated with integral equations approach. In order to show the influence of different type of shape functions of linear charge densities on results in computation, solvers for constant, linear, and cubic spline charge densities over the element are developed. Solution for unknown constant charge densities on elements is developed using BEM and point matching. In order to avoid difficulties in choice of points for point matching [16], Galerkin method is implemented in linear and cubic spline solvers [5].

Since BEM results in fully populated matrix of the system, a compression technique *adaptive cross approximation* (ACA) can be implemented [17–27]. Efficiency of such an approach in computational electromagnetics is presented for a Laplace equation example [22] and scattering problems [23,27].

* Corresponding author.
E-mail address: bojan.trkulja@fer.hr (B. Trkulja).

https://doi.org/10.1016/j.enganabound.2018.03.002
Received 13 September 2017; Received in revised form 5 March 2018; Accepted 5 March 2018
0955-7997/

The contribution of this paper is manifold:

- application of different self-developed BEM solvers to calculate electric field,
- application of ACA method to solve the problem more efficiently,
- benchmarking ACA accelerated BEM with measurements.

2. Calculation method

In order to effectively compute electric field in a substation, only the conductors at known potentials are modeled. The ground is assumed to be on zero potential. The influence of the ground on electric field in the substation is taken into account by the method of images. Since the diameter of HV lines is significantly smaller than the distance to the point where the electric field is computed, thin-wire linear elements in a substation model can be used [9]. The phasor of the electric potential $\dot{\varphi}(\vec{r})$at a point $\vec{r}$due to the unknown phasor of linear charge density $\dot{\lambda}(\vec{r}')$ at $\vec{r}'$ on a thin-wire element is determined by equation [5]:

$$\dot{\varphi}(\vec{r}) = \int\limits_{l'} \frac{\dot{\lambda}(\vec{r}')dl'}{4\pi|\vec{r}-\vec{r}'|}. \tag{1}$$

The unknown function of linear charge density phasor $\dot{\lambda}(\vec{r}')$is computed by boundary element methods. The thin-wires are divided into segments $\Delta l_i'$. The linear charge density phasor on the ith element can be expressed for coefficient K_{ik} of the basis function $s_k$as [9]:

$$\dot{\lambda}_i = \sum_{k=1}^{N_B} K_{ik} s_k. \tag{2}$$

The solvers for computation of electric field are developed for a constant charge density function over the ith segment ($N_B = 1$), a linear charge density function over the ith segment ($N_B = 2$)and a cubic spline charge density function over the ith segment ($N_B = 4$).

2.1. Approximation of the unknown charge density functions with constant charges on thin-wire elements ($N_B = 1$)

For a constant linear charge on the ith straight thin-wire element of the length l, a closed form well-known solution for potential exists [5], and can be described in cylindrical coordinate system as:

$$\dot{\varphi}_i(r,z) = \frac{\dot{\lambda}_i}{4\pi\varepsilon_0} \ln\left(\frac{z+\frac{l}{2}+\sqrt{\left(z+\frac{l}{2}\right)^2+r^2}}{z-\frac{l}{2}+\sqrt{\left(z-\frac{l}{2}\right)^2+r^2}}\right). \tag{3}$$

System of equations is then formed from Eq. (3) for $i = 1, 2, \ldots, N_{seg}$, and solved for unknown $\dot{\lambda}$on segments:

$$\mathbf{C} \cdot \dot{\boldsymbol{\lambda}} = \dot{\boldsymbol{\varphi}}, \tag{4}$$

where **C** is full matrix of coefficients of potentials, derived by the point-matching method [5] at the points on a half of each segment.

Radial component of the electric field $\dot{\vec{E}}_{i,r}$, and axial component $\dot{\vec{E}}_{i,z}$ of the ith thin-wire in a point $\vec{r}$ are calculated from closed form solution [5]:

$$\dot{\vec{E}}_{i,r} = \frac{\dot{\lambda}_i}{4\pi\varepsilon_0 r}\left(\frac{\frac{l}{2}+z}{\sqrt{\left(\frac{l}{2}+z\right)^2+r^2}} + \frac{\frac{l}{2}-z}{\sqrt{\left(\frac{l}{2}-z\right)^2+r^2}}\right), \tag{5}$$

$$\dot{\vec{E}}_{i,z} = \frac{\dot{\lambda}_i}{4\pi\varepsilon_0 r}\left(\frac{r}{\sqrt{\left(\frac{l}{2}-z\right)^2+r^2}} - \frac{r}{\sqrt{\left(\frac{l}{2}+z\right)^2+r^2}}\right). \tag{6}$$

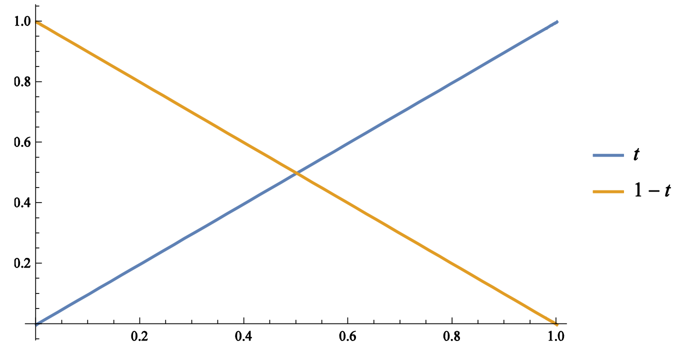

Fig. 1. Linear shape functions.

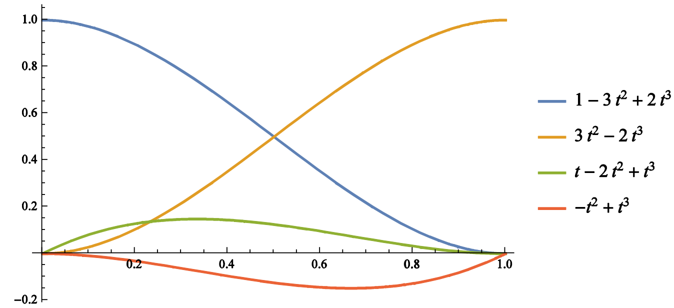

Fig. 2. Cubic spline shape functions.

2.2. Approximation of the unknown charge density functions with linear charges density functions on thin-wire elements ($N_B = 2$)

Linear charge density functions (Fig. 1) on a segment are developed on the dimensionless parameter t, $(0 \geq t \geq 1)$:

$$s_k = \sum_{j=1}^{N_B} a_{kj} t^{j-1}, \quad k = 1, \ldots, N_B \tag{7}$$

Coefficients a_{kj} can be expressed in a matrix:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}. \tag{8}$$

Potential in a point $\vec{r}$ can be calculated combining Eqs. (1) and (2) for linear shape functions to equation:

$$\dot{\varphi}(\vec{r}) = \sum_{i=1}^{N_{seg}} \sum_{k=1}^{N_B} K_{ik} \int\limits_{\Delta l'_i} \frac{s_k(\vec{r}')dl'}{4\pi|\vec{r}-\vec{r}'|}. \tag{9}$$

In order to avoid difficulties in choice of collocation points [16], the coefficientsK_{ik}, representing the linear charge densities values at the end points of segments, are determined from known potentials on the conductors using Galerkin methods [5,14].

Electric field in a point $\vec{r}$is then calculated using:

$$\dot{\vec{E}}(\vec{r}) = \sum_{i=1}^{N_{seg}} \sum_{k=1}^{N_B} K_{ik} \int\limits_{\Delta l'_i} \frac{s_k(\vec{r}')(\vec{r}-\vec{r}')dl'}{4\pi|\vec{r}-\vec{r}'|^3}. \tag{10}$$

2.3. Approximation of the unknown charge density functions with cubic spline charges density functions on thin-wire elements ($N_B = 4$)

Cubic spline charge density functions (Fig. 2.) are determined with Eq. (4), taking into account that the number of basis functions is $N_B = 4$ and the matrix **A** is [9]:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \tag{11}$$