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A regularized boundary element formulation with weighted-collocation and higher-order projection for 3D time-domain elastodynamics

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A B S T R A C T

To advance Time-Domain Boundary Element Methods (TD-BEMs), a generalized direct time-integration solution method for three-dimensional elastodynamics is presented in this paper. On the basis of a general decomposition of time-dependent point-load Green's functions into a singular and regular part, a regularized boundary integral equation for the time domain is formulated and implemented via a variable-weight multi-step collocation scheme that allows for different orders of time projection for the boundary displacements and tractions. The benefits and possibilities of improved performance by suitable collocation weights and the solution projection choices are illustrated via two benchmark finite-domain and infinite-domain problems.

1. Introduction

Boundary element method (BEM) is a powerful numerical method for elastodynamics, as an independent candidate or a component in a hybrid procedure [8] especially in unbounded domain problems. While the approach is commonly implemented in the frequency domain, a sound time-domain boundary element formulation (TD-BEM) is of equal fundamental importance to both theoretical and computational developments. Apart from its clear analytical appeal in being able to handle directly in the time domain fast transient and shock-like dynamic conditions for which a frequency-domain approach will have to face the challenge of determining the system response at very high frequencies, an effective TD-BEM is essential to allow the method to be coupled with a wide variety of mesh-based or meshless methods to realize the best modeling of complex physical problems. To date, a number of TD-BEM formulations have been proposed for 2-D (e.g., [1,6,7,20]) and 3-D problems (e.g., [4,7,11,14,24,25]). While various advances have been found in recent years, there are still basic theoretical and numerical issues that warrant further attention. For example, one common feature among many time-domain BEM formulations is the presence of Cauchy principal values (CPV) of integrals and the jump term as a result of the strong singularity of the point-load traction Green's functions. Although there are schemes for their computation using special quadrature weights, the method of finite part integration [7,15] or rigid-body motion that combines both evaluations [4], they are generally sensitive numerically or limited to specific load and geometric configurations. More fundamentally, numerical instability and accuracy issues in the execution of the time integration have remained the critical challenge to be fully resolved as they tend to be problem- and mesh-specific in time-domain boundary element treatments [8,9,23]. Aimed to mitigate the issue, a number of numerical schemes have been proposed. Examples are the averaged collocation method [19], the ε method [23], the linear-θ method [2], the time-weighting method [10,30,32], and the Galerkin method [31]. While there are alternative avenues to time-domain solution via, for example, the frequency domain through FFT algorithms or the Laplace transform domain with the method of convolution quadrature (CQ) [26,27], the direct time-integration boundary element approach has the fundamental appeal of conceptual simplicity and ease in implementation. To advance this class of methods, a TD-BEM scheme that can be customized parametrically to achieve a higher level of stability and accuracy control should be valuable.

In this paper, a generalized weighted-collocation boundary element method for three-dimensional elastodynamics is presented. On the basis of an analytical decomposition of any time-dependent point-load Green's functions into a singular and regular part [20,28], a regularized boundary integral equation for time-domain analysis is formulated and implemented via a variable-weight multi-step collocation scheme with higher-order temporal projections of the displacement and traction variations. The numerical performance of the formulation using different parametric combinations is tested and compared against more

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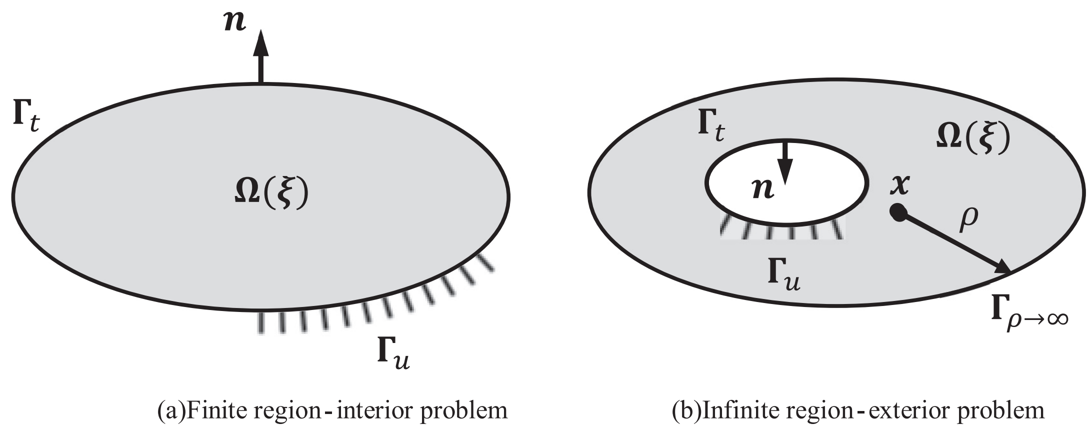

Fig. 1. Elastodynamic boundary value problem.

basic treatments. The possibility of improved performance by an optimal choice of the collocation weights and the order of variable projection is explored via some benchmark finite-domain and infinite-domain problems.

2. Theoretical framework for elastodynamic boundary element method and its regularization

With reference to a Cartesian frame $\{O;\ \xi_1,\xi_2,\xi_3\}$ for a three-dimensional solid in motion whose displacement, Cauchy stress tensor, body force, elasticity tensor and mass density are denoted by $\mathbf{u}(\xi,t)$, $\boldsymbol{\tau}(\xi,t)$, $\mathbf{f}(\xi,t)$, $\mathbf{C}$ and ρ, respectively, in an open regular region Ω bounded by Γ with a quiescent past, it can be shown by Graffi's dynamic reciprocal theorem [29] that the displacement field at a point $\mathbf{x}$ in Ω admits the representation of

$$D(\mathbf{x})\left[u_k(\mathbf{x},t)*g(t)\right] = \int_\Gamma \left[t_i(\boldsymbol{\xi},t)*\hat{U}_i^k(\boldsymbol{\xi},\mathbf{x},t)\right]\mathrm{d}\Gamma_\xi - \int_\Gamma \left[\hat{T}_i^k(\boldsymbol{\xi},\mathbf{x},t;\mathbf{n})*u_i(\boldsymbol{\xi},t)\right]\mathrm{d}\Gamma_\xi + \int_\Omega \left[f_i(\boldsymbol{\xi},t)*\hat{U}_i^k(\boldsymbol{\xi},\mathbf{x},t)\right]\mathrm{d}\Omega_\xi,$$

$$D(\mathbf{x}) = \begin{Bmatrix} 1,\ \mathbf{x}\in\Omega \\ 0,\ \mathbf{x}\notin\Omega \end{Bmatrix} \tag{1}$$

in indicial notation for the interior domain problem depicted in Fig. 1a, with $\mathbf{n}$ being the unit outer normal on the surface Γ and $[a(t)*b(t)]$ the Riemann convolution of the 2 functions $a(t)$ and $b(t)$. With δ_{ik} denoting the Kronecker delta and $\delta(\mathbf{x}-\boldsymbol{\xi})$ the three-dimensional Dirac delta function, $\hat{U}_i^k(\boldsymbol{\xi},\mathbf{x},t)$, $\hat{T}_i^k(\boldsymbol{\xi},\mathbf{x},t;\mathbf{n})$ and $\hat{\Gamma}_{ij}^k(\boldsymbol{\xi},\mathbf{x},t)$ are the displacement, traction and stress Green's functions under a concentrated body force field $\hat{f}_i^k(i=1,2,3)$

$$\hat{f}_i^k(\boldsymbol{\xi},t) = \delta_{ik}\delta(\mathbf{x}-\boldsymbol{\xi})g(t),\ \ t>0,\ \ k=1,2,3, \tag{2}$$

which corresponds to a time-dependent unit point-load in the kth direction acting at a point $\mathbf{x}$ with a magnitude that is described by the arbitrary function $g(t)$. For an unbounded domain Ω that is exterior to the boundary surface Γ, the elastodynamic integral representation is identical to Eq. (1), provided that the unit normal on Γ is directed opposite that for the interior case and the solution satisfies the generalized radiation or regularity condition of

$$\lim_{\rho\to\infty}\int_{\Gamma_\rho}\left(\left[t_i(\boldsymbol{\xi},t)*\hat{U}_i^k(\boldsymbol{\xi},t;\mathbf{x}|g)\right]-\left[\hat{T}_i^k(\boldsymbol{\xi},t;\mathbf{n},\mathbf{x}|g)*u_i(\boldsymbol{\xi},t)\right]\right)\mathrm{d}\Gamma_\xi = 0,\ \forall\mathbf{x}\in\Omega, \tag{3}$$

where Γ_ρ is the spherical outer surface with its radius $\rho\to\infty$ (see Fig. 1b). Taking into account the different orders of singularity of the displacement and traction Green's functions $\hat{U}_i^k$ and $\hat{T}_i^k$, the limiting form of Eq. (1) as $\mathbf{x}\to\mathbf{y}\in\Gamma$ can be stated explicitly as

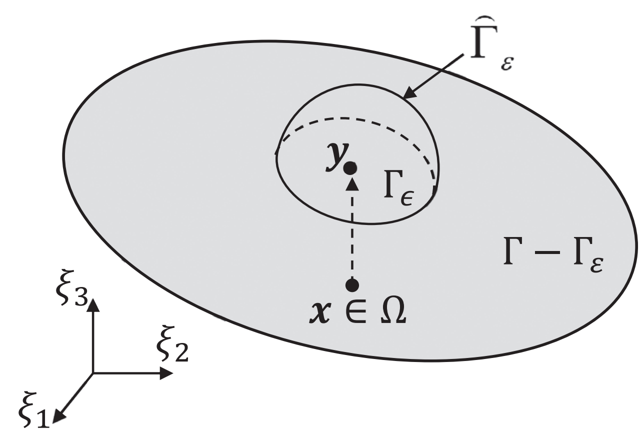

Fig. 2. Formal domain of definition for the jump term c_{ik}.

$$\int_0^t c_{ik}(\mathbf{y},t-\tau)u_i(\mathbf{y},\tau)d\tau + \int_0^t -\!\!\!\!\!\!\int_\Gamma \hat{T}_i^k(\boldsymbol{\xi},\mathbf{y},t-\tau;\mathbf{n})u_i(\boldsymbol{\xi};\tau)d\Gamma_\xi d\tau = \int_0^t\int_\Gamma \hat{U}_i^k(\boldsymbol{\xi},\mathbf{y},t-\tau)t_i(\boldsymbol{\xi},\tau)d\Gamma_\xi d\tau + \int_0^t\int_\Omega \hat{U}_i^k(\boldsymbol{\xi},\mathbf{y},t-\tau)f_i(\boldsymbol{\xi},t)\mathrm{d}\Omega_\xi d\tau,\quad \forall\mathbf{y}\in\Gamma \tag{4}$$

where $-\!\!\!\!\!\!\int_\Gamma$ stands for the Cauchy principal value of the surface integral and

$$c_{ik}(\mathbf{y},t) = \delta_{ik}g(t) + \lim_{\varepsilon\to 0}\int_{\widehat{\Gamma}_\varepsilon}\hat{T}_i^k(\boldsymbol{\xi},\mathbf{y},t,\mathbf{n})d\Gamma_\xi,\ \ \mathbf{y}\in\Gamma, \tag{5}$$

with $\widehat{\Gamma}_\varepsilon$ denoting the small hemispherical surface with a dimension defined by ε and centered at $\mathbf{y}$ (see Fig. 2) so that $\hat{T}_i^k(\boldsymbol{\xi},\mathbf{y},t,\mathbf{n})$ is theoretically non-singular in the limit process.

Despite its classical appeal and adoption, the boundary integral formulation by Eqs. (4) and (5) is not without some common objections. For one, the second integral of Eq. (4) is in terms of its Cauchy principal value whose numerical evaluation is often sensitive as discussed in the Introductin. For non-homogeneous media and non-smooth boundary geometries, a direct evaluation of the jump term c_{ik} in Eq. (5) is also non-trivial generally. To avoid these obtuse computational challenges in a time-domain BEM, an alternative boundary integral equation format which can bypass these issues is proved to be feasible as in [22] and [28]. For this purpose, it is useful to note that dynamic point-load stress Green's function corresponding to Eq. (2) can be decomposed, as