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Modeling of magneto–electro-elastic problems by a meshless local natural neighbor interpolation method

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ABSTRACT

This paper presents a novel numerical procedure based on the meshless local natural neighbor interpolation (MLNNI) method for modeling two-dimensional magneto–electro-elastic solids. As a special case of the generalized meshless local Petrov–Galerkin (MLPG) method, the MLNNI method satisfies the weak form equations locally in polygonal sub-domains which surround each node. The natural neighbor interpolation is used to approximate the unknown fields in numerical simulations and thus only a set of scattered nodes are utilized to represent the problem domain. The usage of three-node triangular FEM shape functions as test functions results in the reduction of the order of integrands in domain integrals. As the constructed shape functions possess a point interpolation property, the essential boundary conditions can be imposed directly without the need of introducing special techniques. Numerical examples for magneto–electro-elastic problems are presented to demonstrate the solutions of the present MLNNI method with other available solutions.

1. Introduction

Magneto–electro-elastic (MEE) composite materials have a wide range of advanced engineering application, because of the mixed characteristics of the piezoelectric, piezomagnetic and magnetoelectric effects [1–3]. Such magnetoelectromechanical coupling makes these materials widely used to construct novel multifunctional devices such as transducers, sensors, and actuators and therefore it is of great importance to simulate magneto–electro-elastic problems in order to determine the field variables. However, it is more difficult to deal with magneto–electro-elastic problems than elasticity ones because of multi-fields coupling involved. Consequently, in recent years, various numerical methods have been developed for modeling the magneto–electro-elastic problems ranging from the finite element method (FEM) [4], the boundary element method (BEM) [2,3], the boundary contour method (BCM) [5], the scaled boundary finite element method (SBFEM) [6,7], and so on.

As an alternative to traditional finite element formulations, meshless methods [8–12] are recently becoming popular and they have been developed rapidly as a class of computational techniques for solving partial differential equations. In the meshless methods, only a set of scattered nodes are required to discretize the problem domain and boundaries and therefore not only the burdensome work of mesh generation can be avoided, but also irregular complex geometries can be described more accurately. Up to now, a wide variety of meshless methods have been developed for analyzing various engineering problems, such as the element free Galerkin (EFG) method [8], the meshless local Petrov–Galerkin method (MLPG) [9,10], the reproducing kernel particle method (RKPM) [13,14], and the meshfree radial point interpolation method [15]. Among these, the MLPG method, which provides the flexibility in choosing the trial and test functions, is one of the most successful meshless methods. Compared to the global weak form method, the MLPG method does not require extra quadrature background cells and is therefore referred to as a truly meshless method. As a result, the MLPG method has been successfully utilized in a wide range of computational problems [16–21]. However, a major shortcoming of the MLPG method is that the computational cost required for evaluating the non-interpolative moving least squares (MLS) approximation is relatively high and special treatments are required to impose the essential boundary conditions. To overcome these shortcomings, Cai and zhu [22] proposed a new technique, the meshless local natural neighbor interpolation (MLNNI) method, which is formulated to unite the advantages of the MLPG method and the natural neighbor interpolation (NNI) [23] with unique properties of its own. The primary advantage of the NNI is that its shape functions possess the Kronecker delta function property and the essential boundary conditions can be imposed in straight forward way as in FEM. Due to these attractive merits, the MLNNI method is applied with great success to the study of transient heat conduction problems [24], dynamics problems [25,26], piezoelectric structures [27], and functionally graded viscoelastic materials [28].

In this paper, the MLNNI method has been developed to solve magneto–electro-elastic problems for the first time. Only a set of scattered nodes are introduced to represent the analyzed domain and there

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is no need to connect the nodes to form closed polygons. Local weak forms over polygonal sub-domains are derived by using the weighted residual method from the coupled governing equations of magneto–electro-elastic problems. The NNI [23] is utilized to approximate spatial variation of displacements, electric and magnetic potentials and the three-node triangular FEM shape functions are used test functions of the weighted residual method. No special treatment is needed to impose the essential boundary conditions and the orders of integrands of domain integrals can be reduced. In the end, numerical examples are solved and comparisons with other available solutions to demonstrate the validity and accuracy of the present technique.

2. Problem definition

If body forces, free electric charges and free magnetic currents are absent, the equilibrium equations of a two-dimensional magneto–electro-elastic problem in a domain Ω bounded by a surface Γ with an outward unit normal vector with the components n_i can be expressed in the following form

$$\sigma_{ij,j} = 0 \tag{1}$$

$$D_{i,i} = 0 \tag{2}$$

$$B_{i,i} = 0 \tag{3}$$

where σ_{ij} are the stress components, D_i represent the electric displacements and B_i denote the magnetic inductions. Note that a comma after a quantity denotes the partial derivative of the quantity.

The strain ε_{ij}, the electric field E_i and the magnetic field H_i can be written in the form

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4}$$

$$E_i = -\Phi_{,i} \tag{5}$$

$$H_i = -\psi_{,i} \tag{6}$$

If direction 3 is taken to be the poling direction and the plane strain state is assumed, the constitutive equations for transversely isotropic magneto–electro-elastic material are given by

$$\begin{aligned}\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix} &= \begin{bmatrix}c_{11} & c_{13} & 0\\ c_{13} & c_{33} & 0\\ 0 & 0 & c_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} - \begin{bmatrix}0 & e_{31}\\ 0 & e_{33}\\ e_{15} & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix}\\ &\quad - \begin{bmatrix}0 & d_{31}\\ 0 & d_{33}\\ d_{15} & 0\end{bmatrix}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\\ &= \boldsymbol{C}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} - \boldsymbol{L}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix} - \boldsymbol{D}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\end{aligned} \tag{7}$$

$$\begin{aligned}\begin{Bmatrix}D_x\\ D_z\end{Bmatrix} &= \begin{bmatrix}0 & 0 & e_{15}\\ e_{31} & e_{33} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} + \begin{bmatrix}h_{11} & 0\\ 0 & h_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix}\\ &\quad + \begin{bmatrix}g_{11} & 0\\ 0 & g_{33}\end{bmatrix}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\\ &= \boldsymbol{G}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} + \boldsymbol{H}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix} + \boldsymbol{A}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\end{aligned} \tag{8}$$

$$\begin{aligned}\begin{Bmatrix}B_x\\ B_z\end{Bmatrix} &= \begin{bmatrix}0 & 0 & d_{15}\\ d_{31} & d_{33} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} + \begin{bmatrix}g_{11} & 0\\ 0 & g_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix}\\ &\quad + \begin{bmatrix}\mu_{11} & 0\\ 0 & \mu_{33}\end{bmatrix}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\\ &= \boldsymbol{R}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} + \boldsymbol{A}\begin{Bmatrix}E_x\\ E_z\end{Bmatrix} + \boldsymbol{M}\begin{Bmatrix}H_x\\ H_z\end{Bmatrix}\end{aligned} \tag{9}$$

where c_{ij}, e_{ij}, d_{ij}, h_{ij}, g_{ij} and μ_{ij} represent the elastic, piezoelectric, piezomagnetic, dielectric, electromagnetic and magnetic constants, respectively. For the plane stress state the constitutive relations can also be obtained from Eqs. (7)–(9) by replacing c_{ij}, e_{ij}, d_{ij}, h_{ij}, g_{ij} and μ_{ij} with $\bar{c}_{ij}$, $\bar{e}_{ij}$, $\bar{d}_{ij}$, $\bar{h}_{ij}$, $\bar{g}_{ij}$ and $\bar{\mu}_{ij}$, which are listed as follows

$$\begin{cases}\bar{c}_{11} = (c_{11}^2 - c_{12}^2)/c_{11}, \bar{c}_{13} = (c_{11} - c_{12})c_{13}/c_{11}, \bar{c}_{33} = (c_{11}c_{33} - c_{13}^2)/c_{11},\\ \bar{c}_{44} = c_{44}\\ \bar{e}_{31} = (c_{11} - c_{12})e_{31}/c_{11}, \bar{e}_{33} = (c_{11}e_{33} - c_{13}e_{31})/c_{11}, \bar{e}_{15} = e_{15}\\ \bar{d}_{31} = (c_{11} - c_{12})d_{31}/c_{11}, \bar{d}_{33} = (c_{11}d_{33} - c_{13}d_{31})/c_{11}, \bar{d}_{15} = d_{15}\\ \bar{h}_{11} = h_{11}, \bar{h}_{33} = (c_{11}h_{33} + e_{31}^2)/c_{11}\\ \bar{g}_{11} = g_{11}, \bar{g}_{33} = (c_{11}g_{33} + e_{31}d_{31})/c_{11}\\ \bar{\mu}_{11} = \mu_{11}, \bar{\mu}_{33} = (c_{11}\mu_{33} + d_{31}^2)/c_{11}\end{cases} \tag{10}$$

The natural mechanical, electrical and magnetic boundary conditions on Γ_t, Γ_q and Γ_b are

$$\sigma_{ij}n_j = \bar{t}_i, \text{on } \Gamma_t \tag{11a}$$

$$D_i n_i = -\bar{\omega}, \text{on } \Gamma_q \tag{11b}$$

$$B_i n_i = -\bar{\eta}, \text{on } \Gamma_b \tag{11c}$$

while the essential mechanical, electrical and magnetic boundary conditions on Γ_u, Γ_p and Γ_a are

$$u_i = \bar{u}_i, \text{on } \Gamma_u \tag{12a}$$

$$\Phi = \bar{\Phi}, \text{on } \Gamma_p \tag{12b}$$

$$\psi = \bar{\psi}, \text{on } \Gamma_a \tag{12c}$$

where $\bar{t}_i$, $\bar{\omega}$, $\bar{\eta}$, $\bar{u}_i$, $\bar{\Phi}$ and $\bar{\psi}$ denote the prescribed values of surface traction, surface charge, surface magnetic induction, displacement, electric potential and magnetic potential, respectively. Note that $\Gamma_u + \Gamma_t = \Gamma_p + \Gamma_q = \Gamma_a + \Gamma_b = \Gamma$.

3. MLNNI formulation for magneto–electro-elastic problems

3.1. Brief of the natural neighbor interpolation

In order to construct shape function, the well-known NNI [23] scheme is utilized in the present study, which is based on the Voronoi diagram and Delaunay tessellation of the domain. Consider a set of distinct nodes $\boldsymbol{N} = \{\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \cdots, \quad \boldsymbol{x}_M\}$ in two-dimensional Euclidean space R^2. The first-order Voronoi diagram of the set $\boldsymbol{N}$ is a unique subdivision of the plane into a series of regions T_I, which is defined as follows [29]

$$T_I = \{\boldsymbol{x} \in R^2 : d(\boldsymbol{x}, \boldsymbol{x}_I) < d(\boldsymbol{x}, \boldsymbol{x}_J) \forall J \neq I\} \tag{13}$$

where $d(\boldsymbol{x}, \boldsymbol{x}_I)$ is the distance between $\boldsymbol{x}$ and $\boldsymbol{x}_I$. The dual of the Voronoi diagram is the Delaunay tessellation.

For purpose of quantifying the neighbours for an inserted point $\boldsymbol{x}$, it is necessary to previously introduce the concept of second-order Voronoi cell T_{IJ}, which can be defined in mathematical terms as

$$T_{IJ} = \left\{\boldsymbol{x} \in R^2 : d(\boldsymbol{x}, \boldsymbol{x}_I) < d(\boldsymbol{x}, \boldsymbol{x}_J) < d(\boldsymbol{x}, \boldsymbol{x}_K) \forall J \neq I \neq K\right\} \tag{14}$$

The Sibson shape function of $\boldsymbol{x}$ with respect to node I is formulated in two dimensions as the ratio of area $A_I(\mathbf{x})$ for T_{xI} and $A(\boldsymbol{x})$ for T_x

$$\phi_I(\boldsymbol{x}) = A_I(\boldsymbol{x})/A(\boldsymbol{x}) \tag{15}$$