Engineering Analysis with Boundary Elements 91 (2018) 14–23

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Construction of special shape functions for triangular elements with one edge lying in the crack front

Guizhong Xie[a,*], Fenglin Zhou[b,*], Dehai Zhang[a], Hongfei Zhai[a], Guangzhen Cui[a]

[a] Henan Key Laboratory of Mechanical Equipment Intelligent Manufacturing, Mechanical and Electrical Engineering Institute, Zhengzhou University of Light Industry, Zhengzhou 450002, Henan, China

[b] College of mechanical engineering, Hunan University of Technology, Zhuzhou 412007, Hunan, China

ARTICLE INFO

Keywords:
Triangular element
Crack front element
Stress intensity factor
Dual boundary element

ABSTRACT

This paper presents a new and general construction method of special triangular elements with one edge lying in the crack front. In our method, principal items which represent the asymptotic properties around the crack front in local coordinate system are constructed. Based on the principal items, two groups of basis functions for crack front elements are introduced. Combining undetermined coefficients and basis functions, two types of crack front elements, which are denoted by low order crack front elements and high order ones, are constructed. Secondly, a displacement extrapolation technique with these newly developed elements is proposed. Applying this technique in the BEM applications for crack problems, numerical difficulties, which arise when two vertexes of the assisted line both locate inside the element, can be circumvented conveniently. Finally, two types of elements are employed in the dual boundary element codes for crack problems. Numerical examples are presented to verify the three types of elements. Results show that the stress intensity factors computed by employing these three developed elements are all very accurate compared with the existing results.

1. Introduction

In many fields of finite element [1–5] and boundary element analysis [6–12], elements which are equipped with special shape functions are usually necessary to obtain accurate results, especially in the solution of potential problems, in which the sharp corners and edge singularities are involved, and crack problems with stress field singularity [7,9,10–14]. Compared with standard isoparametric element, the specially defined shape functions in those specially constructed elements usually possess the asymptotic properties around the sharp corners and edges, or the crack tips. This paper focuses attention on the construction of triangle elements with singularities for the crack fronts.

Various works on singular finite elements can be found. In those singular elements, the derivatives of the shape functions at a specific node (one of its vertices) possess singularities [1–5]. Employing the singular finite elements, the desirable singular behavior can be successfully captured. Thus, the results with high accuracy can be obtained. In the pioneer work of Blackburn, the 3, 4 and 6-node special triangular elements were constructed [1]. With the help of these special elements, the displacement components possess $\sqrt{r}$ asymptotic properties, where r is the distance between the position coordinates and the singular node. Henshell and Shaw constructed a type of special eight-node quadrilateral element [2]. Moreover, Barsoum found that the 6-node triangular element can be obtained by collapsing one side of the quadrilateral isoparametric element [3]. These special triangular elements were constructed and used as elastic and plastic crack tip element. Hibbitt reviewed the singular properties of special constructed isoparametric elements [4]. In all those special elements above, however, only the asymptotic behavior of some specific node could be captured. In practical cases, for example in boundary element analysis, special shape functions which possess the asymptotic behaviors of one specific edge are also necessary. In this paper, we will construct the special shape functions based on the asymptotic behavior of displacement components around one specific edge of a triangular element. To model the singularities arise around one specific node, the 6-node triangular element can be found in reference [5].

The special elements employed for boundary element analysis have benefited from the singular finite elements, such as quarter-point element by modifying the position of the element nodes [14]. Mi Y and M.H. Aliabadi have developed a family of 8-node discontinuous crack-tip elements and applied them in the 3D dual boundary element method (DBEM) for crack problems [7]. H. kebir et al have developed a 3-node discontinuous crack-tip element for crack problems and applied it to bolted joints [15]. Pan and Yuan proposed a novel type of singular element through weight coefficient and successfully applied this type singular element in the DBEM [8]. Xie et al. extended the pioneer

* Corresponding author.
E-mail addresses: xieguizhong@zzuli.edu.cn (G. Xie), zhoufl@hnu.edu.cn (F. Zhou).

https://doi.org/10.1016/j.enganabound.2018.03.008
Received 31 January 2018; Received in revised form 3 March 2018; Accepted 13 March 2018
Available online 27 March 2018

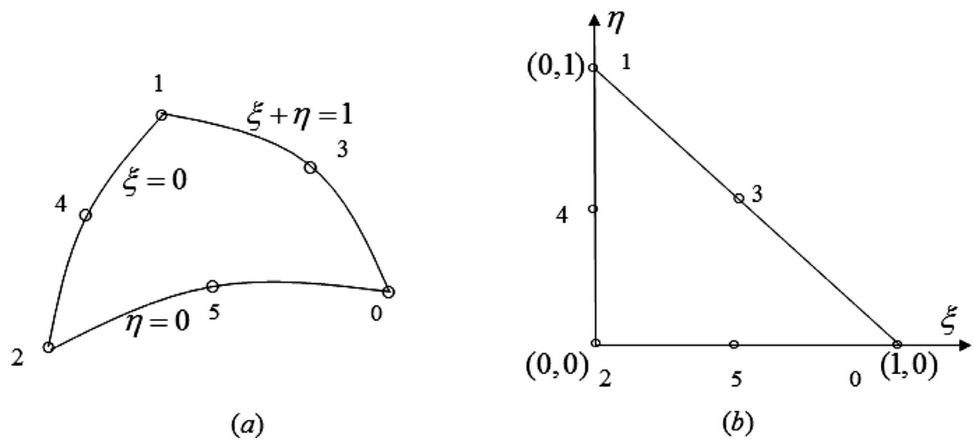

Fig. 1. 6-node continuous triangular element.

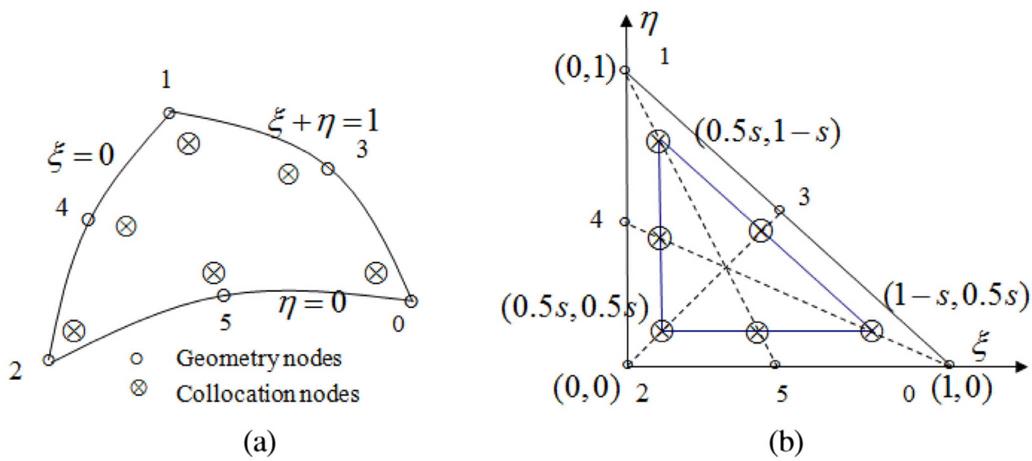

Fig. 2. 6-node discontinuous triangular element.

work of Mi and Aliabadi to a family of 9-node discontinuous crack-tip elements [16,17]. Li et al. proposed three types of three- and nine-node discontinuous crack-tip elements through undetermined coefficients methods [18]. In our method, triangular crack front elements are developed and implemented into the DBEM. Thus, this paper pays no attention to details of the DBEM. More details about DBEM which using the traction boundary integral equation as supplementary on the crack surfaces can be found in references [6,17–20].

This paper presents novel and general special elements in triangular geometry based on discontinuous triangular elements. Firstly, the asymptotic behaviors of displacement components around the specific edges (the edge lies in the crack front) are deduced in the isoparametric space to find the principle items in local coordinate system. Secondly, based on the principal items, two groups of special basis functions can be defined. Using the given basis functions, the shape functions which have the desired singular properties can be constructed. Then a displacement extrapolation technique will be developed with employing the two types of special elements. Finally, two types of crack front elements are applied in 3D DBEM for crack problems. Numerical examples will be given to verify the accuracy and efficiency of the presented technique.

The content of our paper is as follows. In Section 2, 6-node continuous and discontinuous triangular elements are introduced. Construction of discontinuous triangular crack front elements is detailed in Section 3. Section 4 introduces the displacement extrapolation technique. Numerical examples are presented in Section 5. The paper ends with conclusions in Section 6.

2. *6-node continuous and discontinuous triangular elements*

In the continuous triangular element as shown in Fig. 1, the displacement at an arbitrary point in the element can be interpolated by displacements on these six nodal points. The shape functions defined in the 6-node triangular element are as follows [5]:

$$
\begin{aligned}
&N_0 = \xi(2\xi - 1) \; N_3 = 4\xi\eta \\
&N_1 = \eta(2\eta - 1) \; N_4 = 4\eta(1 - \xi - \eta) \\
&N_2 = (1 - \xi - \eta)(2(1 - \xi - \eta) - 1) \; N_5 = 4\xi(1 - \xi - \eta)
\end{aligned} \tag{1}
$$

In Fig. 1, since the geometric nodes coincide with collation nodes, the shape functions for geometric quantities and physical quantities are also identical. In the DBEM for crack problems, however, the continuity requirement of the field variables should be satisfied to guarantee the existence of Cauchy and Hadamard principle value integrals. Thus, instead of continuous elements, discontinuous elements are most widely used. In the 6-node discontinuous triangular element, the collocation nodes are usually distributed inside the element as illustrated in Fig. 2, where s is a position parameter of the collocation nodes. The shape functions for geometric quantities in Fig. 2 are the same as those of the continuous triangular element, while the shape functions for the physical quantities are different from those in continuous triangular elements. In order to show their differences, we use N^i_{geo} for the denotation of shape functions about geometric quantities and N^i_{coll} for shape functions about physical quantities, where $N^i_{geo} = N_i, i = 0..5$,

$$
\begin{aligned}
N^0_{Coll} &= -\frac{[\xi - (0.5 - 0.25s)](\xi - 0.5s)}{(0.5 + 0.75s)(1 - 1.5s)}, \\
N^1_{Coll} &= -\frac{[\eta - (0.5 - 0.25s)](\eta - 0.5s)}{(0.5 + 0.75s)(1 - 1.5s)}, \\
N^2_{Coll} &= \frac{(1.0 - 0.5s - \xi - \eta)(0.5 + 0.25s - \xi - \eta)}{(1 - 1.5s)(0.5 - 0.75s)},
\end{aligned}
$$