Engineering Analysis with Boundary Elements 000 (2017) 1–20

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A naturally stabilized nodal integration meshfree formulation for carbon nanotube-reinforced composite plate analysis

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ARTICLE INFO

Keywords:
Naturally stabilized nodal integration
Moving Kriging interpolation
Meshfree method
Carbon nanotube-reinforced composite plates

ABSTRACT

Naturally stabilized nodal integration (NSNI) meshfree formulations associated with the higher-order shear deformation plate theory (HSDT) are proposed to analyze bending and free vibration behaviors of carbon nanotube-reinforced composite (CNTRC) plates. An extended rule of mixture is used to compute the effective material properties of CNTRC plates. The uniform and functionally graded distributions of carbon nanotube (CNTs) via the plate thickness are studied. In the present approach, gradient strains are directly computed at nodes similar to the direct nodal integration (DNI). Outstanding features of the current approach are to alleviate instability solutions in the DNI and to significantly decrease computational cost as compared to the traditional high-order Gauss quadrature scheme. Discrete equations for bending and free vibration analyses are obtained by variational consistency in the Galerkin weak form. Enforcing essential boundary conditions is completely similar to the finite element method (FEM) due to satisfying the Kronecker delta function property of moving Kriging integration shape functions. Numerical validations with various complex geometries, stiffness ratios, volume fraction of CNTs and boundary conditions are given to show the efficiency of the present approach.

1. Introduction

In recent years, composites materials reinforced carbon nanotubes (CNTs), namely CNT-reinforced composites (CNTRC), have been recently known in various applications of aerospace, aeronautics, marine, and civil areas, since their excellent mechanical, electrical and thermal features [1]. Practically, four types of distributions of CNTs across the plate thickness, including of UD, FGV, FGO and FGX, are considered. For instance, UD expresses a uniform distribution while FGV, FGO and FGX denote three different functionally graded distributions, as shown in Fig. 1. The CNTRC plate is called the functionally graded carbon nanotube reinforced composite (FG-CNTRC) plate when CNTs are distributed by types of functionally graded via the plate thickness. These materials are usually used in engineering applications as beam, plate and shell structures. Therefore, to accurately predict the behavior of these structural components, the understandings of displacements, stresses and free vibration natural frequency are highly required.

It has been found in the literature that, computational theories addressing to the CNTRC plate structure can be classified into two primary groups according to the three-dimensional (3D) elasticity and the 2D plate models. The full 3D elasticity model does not need hypotheses for stress and deformation distributions and is taken as a complete 3D solid. The actual behavior of plates can be predicted but it has high computational cost. In addition, it has many difficulties for any problems with complex geometries and arbitrary boundary conditions. Several solutions of a 3D elasticity model for simple cases of CNTRC plates were investigated in [2–5]. The 2D plate model includes the classical plate theory (CPT), the first order shear deformation theory (FSDT), the higher order shear deformation theory (HSDT) and the quasi-3D shear deformation theory. The first one [6,7] showed the inaccurate results for CNTRC plates due to skipping effects of transverse shear strains. The second one assumes a constant of transverse shear stresses across the plate thickness so a shear correction factor needs to additionally match the strain energy derived from FSDT solution with those from the 3D

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https://doi.org/10.1016/j.enganabound.2017.10.018
Received 29 May 2017; Received in revised form 25 August 2017; Accepted 21 October 2017
Available online xxx
0955-7997/

elasticity solution. Roughly speaking, it is difficult to determine an optimized value for shear correction factors which depend on the material properties, geometries and boundary conditions of problems. Several results are presented for analysis of CNTRC single plates based on the FSDT such as static and free vibration [8,9], buckling [10,11], elasto-dynamic [12], large deformation [13,14] and post-buckling [15]. In addition, the FSDT is used for analysis of CNTRC laminated plates [16–19]. The third one involved (1) polynomial forms as the third order shear deformation theory (TSDT) [20,21], the fifth order shear deformation theory (FiSDT) [22], the seventh order shear deformation theory [23]; (2) non-polynomial forms as the trigonometric shear deformation theory [24–26], the inverse trigonometric shear deformation theory [27], the exponential shear deformation theory (ESDT) [28,29] and so on. The HSDT has been applied for CNTRC single plates [30–36] and CNTRC sandwich plates [37,38]. The last one is called the quasi-3D theory, additionally taking into accounts for stretching effect in plate thickness direction and using the fully 3D material matrix in the relation between stress and strain. This theory has also been developed for CNTRC sandwich plates [39,40]. The higher order plate theories mentioned requires the C^1-continuity of strain field and contained slope components related to the derivation of the transverse displacement. It is really difficult to impose boundary condition for these slope components which are not considered as the approximation variables. Therefore, an alternative theory as C^0-type higher order shear deformation theory should be used.

There are some solution procedures for analysis of the CNTRC plates in the literature. They can be divided into three following types: analytical approach [41], semi-analytical approach [6] and numerical method (e.g., finite element [8], meshfree [17] and isogeometric analysis [34,42], etc…). The first and second approaches are appropriate for simple geometries and boundary conditions. For arbitrary geometries and boundary conditions, the last approach is the best choice. In addition, these numerical methods usually use strong [43–46] or weak [8,17] forms to derive essential equations. In this paper, we further develop the meshfree method based on the Galerkin weak form for analysis of the CNTRC plates. For this approach, a choice of efficient quadrature techniques is necessary to exactly integrate rational shape functions into meshfree methods. Generally speaking, there are two popular types of integration in the Galerkin weak form: Gauss quadrature and nodal integrations. In order to compute the Gauss quadrature integration, background cells similar to FEM are required. Due to rational shape functions, the high-order Gauss quadrature scheme is performed to obtain stable and accurate solutions but it consumes much computational time. On the contrary, solutions may not be converged and stabilized [47,48] when using the lower-order Gauss quadrature. To reduce computational cost, the nodal integration technique is an appropriate choice. However, the directly nodal integration gives unstable solutions due to rank deficiency of approximated matrices. To achieve stable solutions, several nodal integration schemes are presented for the meshfree method based on in the Galerkin weak such as a stabilized conforming nodal integration (SCNI) by Chen et al. [47], a modified SCNI by Hillman et al. [49], a least-squares method by Beissel and Belytschko [50], a Taylor series expansion of the displacement fields by Nagashima [51] and Liu et al. [52], a Taylor expansion combined with displacement smoothing by Wu et al. [53] and so on. Recently, Hillman and Chen [54] have proposed a naturally stabilized nodal integration (NSNI) technique for a reproducing kernel particle method. This technique is a naturally implicit gradient expansion. This paper exploits the advantages of the NSNI for the meshfree method based on moving Kriging integration shape functions for bending and free vibration analyses of carbon nanotube-reinforced composite plates.

The outline of the paper is described as follows. In Section 2, basic equations of CNTRC plates are presented. In Section 3, the NSNI technique for CNTRC plates is introduced. Section 4 shows the designing of geometries and imposing of essential boundary conditions. Numerical results are illustrated in Section 5. Section 6 gives some concluding remarks.

Table 1
CNTs efficiency parameters for different values of volume fractions [56].

V_{CNT}^{*}	η_1	η_2	η_3
0.11	0.149	0.934	0.934
0.14	0.150	0.941	0.941
0.17	0.149	1.381	1.381

2. Basic equations

2.1. Problem description

Let us consider a carbon nanotube-reinforced composite plate made of a polymeric matrix reinforced with single walled carbon nanotube (SWCNT). As shown in Fig. 1, there are four types of distributions of CNTs across the plate thickness consisting of UD, FG-V, FG-O and FG-X used for the present work, in which UD describes the uniform distribution and FG-V, FG-O and FG-X define the three different functionally graded distributions. The distributions of CNTs along the plate thickness direction according to Fig. 1 are expressed by

$$\begin{cases} V_{CNT} = V_{CNT}^{*} & (UD) \\ V_{CNT}(z) = \left(1 + \frac{2z}{h}\right) V_{CNT}^{*} & (FGV) \\ V_{CNT}(z) = 2\left(1 - \frac{2|z|}{h}\right) V_{CNT}^{*} & (FGO) \\ V_{CNT}(z) = 2\left(\frac{2|z|}{h}\right) V_{CNT}^{*} & (FGX) \end{cases} \tag{1}$$

where

$$V_{CNT}^{*} = \frac{w_{CNT}}{w_{CNT} + \left(\rho^{CNT}/\rho^{m}\right) - \left(\rho^{CNT}/\rho^{m}\right) w_{CNT}} \tag{2}$$

in which w_{CNT}, ^{m} and ρ^{CNT} are the mass fraction of the carbon nanotube, the density of the matrix and the density of the CNTs, respectively.

There are two common schemes following the extended rule of mixture [55,56] and Eshelby-Mori-Tanaka [57] to estimate the effective material properties of CNTs-reinforced materials. These properties are highly dependent on the structure of CNTs. For the simplification of computation, the extended rule of mixture is used. According to this mixed rule, the effective material properties of FG-CNTRC plate are denoted as follows [56]

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \tag{3}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^m} \tag{4}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^m} \tag{5}$$

where E_{11}^{CNT}, E_{22}^{CNT} and G_{12}^{CNT} are the Young's moduli and shear modulus of CNTs, respectively. E^m and G^m are the Young's moduli and shear modulus of the isotropic matrix, respectively. V_{CNT} and V_m are the volume fractions of the carbon nanotubes and matrix, respectively, and they are taken by $V_{CNT} + V_m = 1$. In addition, three efficiency parameters of the CNTs as $\eta_j(1,2,3)$ are given to explain for the scale-dependent material properties, as listed in Table 1. The parameters of the effective properties obtained from the molecular dynamic (MD) simulations following the rule of mixture [58].

Similarly, Poisson's ratio (υ_{12}) and the density (ρ) of the FG-CNTRC plates are computed by the rule of mixture as

$$\upsilon_{12} = V_{CNT}^{*} \upsilon_{12}^{CNT} + V_m \upsilon^m \tag{6}$$

$$\rho = V_{CNT} \rho^{CNT} + V_m \rho^m \tag{7}$$

where υ_{12}^{CNT} and ν^m are Poisson's ratios of the CNTs and matrix, respectively.