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Radial basis reproducing kernel particle method for piezoelectric materials

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ABSTRACT

To reduce the negative effect of different kernel functions on calculating accuracy, the radial basis function (RBF) is introduced into the reproducing kernel particle method (RKPM), and the radial basis reproducing kernel particle method (RRKPM) is proposed, the corresponding governing equations are derived. The RRKPM is more efficient to solve the local problem domain, and can improve the accuracy and stability of the RKPM. Then the RRKPM is applied to the numerical simulation of piezoelectric materials, the corresponding formulae for piezoelectric materials are derived. The numerical results illustrate the proposed method is more stable and accurate than the RKPM.

1. Introduction

Piezoelectric materials have electro–mechanical coupling characteristics, and have been widely used in sensors and actuators [1,2]. Sensors and actuators are the main part of the electronic control system, and their accuracy, sensitivity and stability have a great effect on the performance of the electronic control system. So it is important to simulate the sensing and driving properties of piezoelectric material.

When the piezoelectric material is subjected to a machanical deformation, the voltage is generated in the material. Likewise, if a voltage is applied to the piezoelectric material, displacement is generated in the material. These two phenomena are called the direct piezoelectric effect and the indirect piezoelectric effect, respectively. Based on the direct piezoelectric effect and the indirect piezoelectric effect, various instruments were designed, such as pressure sensors, buzzers, microphones, ultrasonic rotary motors, piezoelectric filters, infrared detectors [3], etc.

At present, the numerical simulation is the main method to research piezoelectric material because it can quickly and efficiently solve the domain. Numerical simulation methods mainly include finite element method (FEM) [4,5], boundary element method (BEM) [6], finite difference method (FDM) [7] and meshless method (MM) [8,9], etc. The FEM is the most general numerical method in engineering calculation, but the FEM needs to spend a lot of time to achieve the refinement of the meshes when solving the local domain problem of complex structure precisely, besides, the adaptive analysis is difficult. The BEM needs to integrate domain to solve the problem, it shows strong singularity near the singular point and difficult to calculate. The FDM is very complicated in the handling of irregular areas, and unsuitable for dealing with engineering problems which have complex boundary conditions.

The meshless method has developed in recent years, and has the advantages of quick calculation and high precision when it is used to solve problem of piezoelectric material [10]. The meshless method uses a series of appropriate scattered nodes to analyze the problem domain and boundary, and constructs the approximate function in the problem domain. When the problem domain is needed to solve precisely, it only needs to increase the number of nodes. Compared with the FEM, the meshless method does not depend on the meshes and the relationships of nodes, the amount of storage can be saved.

There are a variety of meshless methods, such as the reproducing kernel particle method (RKPM) [11–13], the radial basis function method (RBF) [14,15], the element-free Galerkin method (EFGM) [16–19], the finite point method (FPM) [20,21], the partition of unity method (PUM) [22,23], the polynomial point interpolation method (PPIM) [24,25], the moving least-squares method (MLS) [26–28], the local Petrov–Galerkin method (LPGM) [29–32], the smooth particle hydrodynamics (SPH) [33,34], the boundary integral equation method (BIEM) [35,36], the Hermite radial point interpolation (Hermite RPI) [37–40] and the meshless manifold method (MMM) [41,42].

The core of the RKPM is that a series of the reproducing kernel functions are constructed by using approximate functions in the problem domain. The RKPM has been used to solve displacements and electric potential of piezoelectric materials because of its fast convergence speed. But in the process of solving the domain, the different kernel function has a negative effect on the calculating accuracy. In order to eliminate the defect of kernel function in aspect of calculating accuracy, the RBF is introduced to the RKPM, and the radial basis reproducing kernel particle

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method (RRKPM) is presented in this paper. The accuracy and stability of the RRKPM are proved by the numerical examples of piezoelectric bimorph cantilever beam and the piezoelectric strip.

2. The governing equations of piezoelectric materials

In the x–z plane, the constitutive equations of the piezoelectric materials can be analyzed from two respects of the strain and the electric field

$$\boldsymbol{\sigma} = \boldsymbol{c}^E \boldsymbol{\varepsilon} - e\boldsymbol{E} \tag{1}$$

$$\boldsymbol{D} = \boldsymbol{e}\boldsymbol{\varepsilon} + \boldsymbol{\xi}^{\varepsilon} \boldsymbol{E} \tag{2}$$

where $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$, $\boldsymbol{E}$ and $\boldsymbol{D}$ are the strain tensor, the stress tensor, the electric field tensor and the electric displacement tensor, respectively. $\boldsymbol{e}$, $\boldsymbol{c}^E$ and ξ^{ε} are the piezoelectric constant matrix, the elastic stiffness matrix and the dielectric constant matrix, respectively. The superscripts ε and E represent the coefficients measured under constant electric and stress conditions.

The relationship between strain and displacement can be expressed as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

Where i, j represent x, z, respectively, and the above condensed relationship can be rewritten as

$$\varepsilon_x = \varepsilon_{xx} = u_{,x} \tag{4}$$

$$\varepsilon_z = \varepsilon_{zz} = w_{,z} \tag{5}$$

$$\gamma_{xz} = 2\varepsilon_{xz} = u_{,z} + w_{,x} \tag{6}$$

where u and w represent the displacements in the x and z-directions, respectively. Commas followed by indices represent partial differentiation with regard to the respective coordinate (i.e. $u_{,x} = \partial u/\partial x$).

The relationship between electric field and electric potential can be given as

$$E_i = -\phi_{,i} \tag{7}$$

The mechanical equilibrium equations for piezoelectric materials is

$$\sigma_{ij,j} = 0 \tag{8}$$

The electrical equilibrium equation for piezoelectric materials is

$$D_{i,i} = 0 \tag{9}$$

Substituting Eqs. (1), (2), (4)–(6) and (7) into equilibrium Eqs. (8) and (9), the equilibrium equations can be written in the forms of displacement and electric potential.

$$\begin{bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & c_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{bmatrix} - \begin{bmatrix} 0 & e_{31} \\ 0 & e_{33} \\ e_{15} & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_z \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} D_x \\ D_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & e_{15} \\ e_{31} & e_{33} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{bmatrix} + \begin{bmatrix} \xi_{11}^{\varepsilon} & 0 \\ 0 & \xi_{33}^{\varepsilon} \end{bmatrix} \begin{bmatrix} E_x \\ E_z \end{bmatrix} \tag{11}$$

The governing mechanical equilibrium equations of the piezoelectric materials are

$$c_{11}u_{,xx} + c_{55}u_{,zz} + (c_{13} + c_{55})w_{,xz} + (e_{31} + e_{15})\phi_{,xz} = 0 \tag{12}$$

$$(c_{13} + c_{55})u_{,xz} + c_{33}w_{,zz} + c_{55}w_{,xx} + e_{33}\phi_{,zz} + e_{15}\phi_{,xx} = 0 \tag{13}$$

The governing electrical equilibrium equations of piezoelectric materials is

$$(e_{31} + e_{15})u_{,xz} + e_{15}w_{,xx} + e_{33}w_{,zz} - \xi_{11}^{\varepsilon}\phi_{,xx} - \xi_{33}^{\varepsilon}\phi_{,zz} = 0 \tag{14}$$

3. The RRKPM of piezoelectric materials

The approximate displacement function $w^h(x,z)$ can be expressed as the combination of the radial basis functions constructed by internal nodes (n_m) and the reproducing kernel functions constructed by the all nodes (n) in local supporting domain.

$$w^h(x,z) = \sum_{k=1}^{n} R_k(x,z)c_k + \sum_{i=1}^{n_m} R_i^m(x,z)a_i \tag{15}$$

where a_i and c_k are undetermined coefficients; n is the number of all nodes in the local supporting domain; n_m is the number of internal nodes; R_i^m is a radial basis function constructed by internal nodes; R_k is a reproducing kernel function constructed by the all nodes.

In Eq. (15), the radial basis function R_i^m can be expressed as a class of functions, $r_i = \sqrt{(x-x_i)^2 + (z-z_i)^2}$, and r_i depends only on the distance between evaluation nodes (x, z) and the nodes (x_i, z_i)

$$R_i^m(x,z) = \left(1 - \frac{r_i}{\delta}\right)^5 \left(8 + 40\frac{r_i}{\delta} + 48\frac{{r_i}^2}{\delta^2} + 25\frac{{r_i}^3}{\delta^3} + 5\frac{{r_i}^4}{\delta^4}\right) \tag{16}$$

where δ is the shaped parameter, the reproducing kernel function can be given as

$$R_k(x,z) = c_k(x - x_I, z - z_I)w(r_k)R_k(x_I, z_I)\Delta V_I \tag{17}$$

where x_I and z_I are the coordinates of node I, $R_k(x_I,z_I)$ is the unknown parameter of node I, ΔV_I is the influence domain of node I.

$$c_k(x - x_I, z - z_I) = \boldsymbol{b}_k^T(x,z)\boldsymbol{p}_k(x - x_I, z - z_I) \tag{18}$$

The coefficient matrix is

$$\boldsymbol{b}_k(x,z) = [\, b_1(x,z) \quad b_2(x,z) \quad \cdots \quad b_6(x,z)\,]^T \tag{19}$$

where $\boldsymbol{b}_k(x,z)$can be obtained from the conditions of the reproducing kernel approximation.

The polynomial basis function is

$$\boldsymbol{p}_k^T(x - x_I, z - z_I) = [\,1,\ x{-}x_I,\ z{-}z_I,\ (x{-}x_I)^2,\ (x{-}x_I)(z{-}z_I),\ (z{-}z_I)^2] \tag{20}$$

In Eq. (17), $r_k = d_k/d_{mk}$, $d_k = \sqrt{(x-x_k)^2 + (z-z_k)^2}$, d_{mk} is the radius of the supporting domain defined by the reproducing kernel function at node (x_k,z_k).

In order to analyze the influence of the kernel function $w(r_k)$ on calculating accuracy, the kernel function $w(r_k)$ is taken in the following two forms as

$$w_1(r_k) = \begin{cases} 2/3 - 4{r_k}^2 + 4{r_k}^3 & r_k \le 1/2 \\ 4/3 - 4r_k + 4{r_k}^2 - 4{r_k}^3/3 & 1/2 < r_k \le 1 \\ 0 & r_k > 1 \end{cases} \tag{21}$$

$$w_2(r_k) = \begin{cases} 1 - 2r^2 & r_k \le 1/2 \\ 2(1-r)^2 & 1/2 < r_k \le 1 \\ 0 & r_k > 1 \end{cases} \tag{22}$$

Eq. (15) can be written in the matrix form as

$$w^h(x,z) = \boldsymbol{B}^T\boldsymbol{a}_0 \tag{23}$$

where $\boldsymbol{B}$ is the basis function vector, and can be given as

$$\boldsymbol{B}^T = [R_1 \quad R_2 \quad \cdots \quad R_n \quad R_1^k(x,z) \quad R_2^k(x,z) \quad \cdots \quad R_{n_m}^k(x,z)] \tag{24}$$

The coefficient vector $\boldsymbol{a}_0$ can be expressed as

$$\boldsymbol{a}_0^T = \{a_1 \quad a_2 \quad \cdots \quad a_n \quad c_1 \quad c_2 \quad \cdots \quad c_{n_m}\} \tag{25}$$

In Eq. (15), the coefficients a_i and c_k can be determined by the function values of all nodes in the supporting domain.

The function values for all nodes in the supporting domain can be deduced

$$w(x_I, z_I) = \sum_{k=1}^{n} R_k(x_I, z_I)c_k + \sum_{i=1}^{n_m} R_i^m(x_I, z_I)a_i \tag{26}$$