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On nonlinear analysis by the multipoint meshless FDM

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ABSTRACT

The main objective of this paper is to present an attempt of an application of the recently developed higher order multipoint meshless FDM in the analysis of nonlinear problems. The multipoint approach provides a higher order approximation and improves the precision of the solution. In addition to improved solution quality, the essential feature of the multipoint approach is its potentially wide ranging applicability. This is possible, because in both the multipoint and standard meshless FDM, the difference formulas are generated at once for the full set of derivatives. Using them, we may easily compose any required FD operator. It is worth mentioning that all derivative operators depend on the domain discretization rather than on the specific problem being analysed. Therefore, the solution of a wide class of problems including nonlinear ones, may be obtained with this method.

The numerical algorithm of the multipoint method for nonlinear analysis is presented in this paper. Results of selected engineering benchmark problems – deflection of the ideal membrane and analysis of large deflection of plates using the von Karman theory – are considered.

1. Introduction

The most popular method for numerical analysis of the engineering problems of last decades is the Finite Element Method (FEM) [1]. However, besides the FEM, many other methods have been developed over time. To avoid the difficulties encountered in the traditional FE approach, such as the sometimes troublesome process of mesh generation in the case of complex geometries, remeshing, and mesh distortions in large deformation problems – alternative methods of discrete analysis have also been developed. In the recent years, among them various so called meshless methods and isogeometric methods [2], which use CAD software for geometric discretization, were appeared.

As opposed to the finite element approach, the meshless methods may deal with a totally irregular cloud of nodes only rather than with structure composed of elements. Due to the arbitrarily distributed nodes without any imposed structure (such as splitting domain into finite elements, mapping restrictions, or mesh regularity), the local changes of discretization, e.g. inserting, deleting, and moving particular nodes, may be applied without difficulties. Various meshless methods [3] differ from each other in the process of generation of the unknown function local approximation. Such approximation is performed around the nodes rather than between them, as in the FEM.

Both approaches, the meshless methods, as well as FE method have of course their own advantages and disadvantages. The appropriate optimal choice depends on the object geometry, the type of problem, and many other important factors.

The oldest and possibly one of the most developed meshless method is the Meshless Finite Difference Method (MFDM) [4,5]. The Moving Weighted Least Squares (MWLS) approximation technique [5], based on the nodes configuration called the FD star or stencil, is applied in this method.

The innovative higher order extension of the MFDM, namely the Multipoint meshless finite difference method [6] – has been recently developed by the authors for analysis of boundary value (b.v.) problems. The method formulation, following the original Collatz [8] multipoint FD concept (outlined only for the regular mesh and strong formulation), has been modified and extended to the multipoint MFDM. The new multipoint method is based on the moving weighted least squares approximation technique instead of the polynomial interpolation proposed by Collatz. Moreover, arbitrarily irregularly distributed nodes, as well as strong, or various weak, or mixed (local–global) formulations of the b.v. problems may be used here.

The idea of the multipoint approach [6] is based on raising the approximation order of unknown function by using a combination of searched function values together with a combination of additional degrees of freedom (d.o.f.) at all stencil nodes. The known values of the considered equation right hand side (the specific case), or unknown function derivatives (the general case) may be used as the additional d.o.f. Such approach provides higher solution order and, as a consequence, better quality without increasing the mesh density. It uses the same FD stars that are needed to generate the FD operators in the standard MFDM approach.

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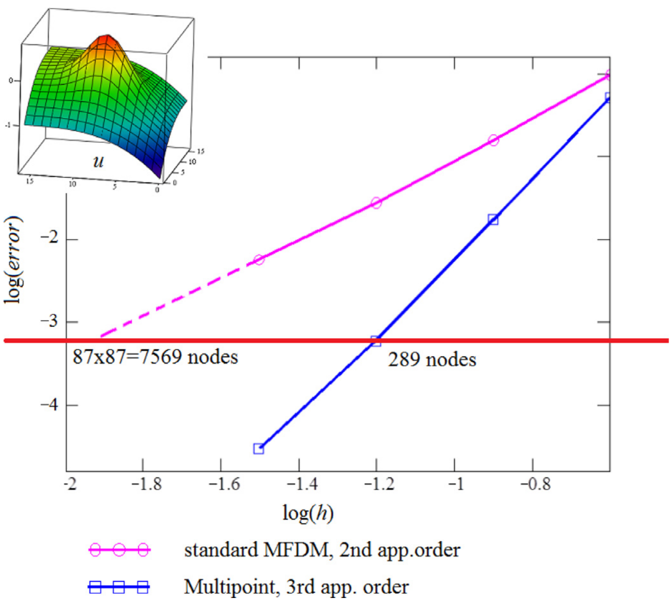

Fig. 1. Solution convergence for the multipoint method and the standard MFDM (Poisson's b.v. problem).

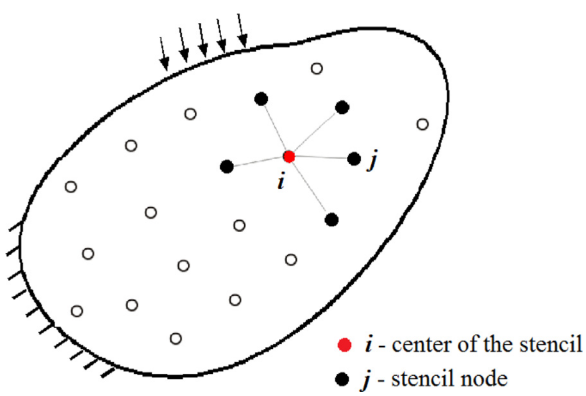

Fig. 2. MFD star for arbitrarily distributed nodes in the domain.

There are numerous advantages of the multipoint MFDM approach. Due to its higher order approximation, the method allows for using decreased number of nodes (Fig. 1). The same solution quality may be achieved either on a coarse mesh using the high order approximation approach (*p*-type), or by solving a problem on the very dense mesh (*h*-type), which often requires remeshing with high computational cost. In addition to solution quality improvement, the proposed multipoint method, like the standard MFDM, is especially convenient for nonlinear analysis, due to fast generation of updated stiffness matrices and evaluation of all unknown derivatives in terms of only searched nodal function values.

The MFD-based methods allow for analysis of a wide class of b.v. problems because they generate MFD operators rather for full range of particular derivatives of required order, than any specific operator like in the FEM. In this way, any kind of differential operator needed may be composed of those particular ones. Moreover, the FD solution approach based on the MWLS approximation provides the derivative operators without any special additional cost. In this case the MFD operators depend on discretization of the problem domain only. This fact is advantageous from the point of view of the calculation efficiency, especially for engineering tasks, where often problem formulation may be changed at whole domain or part of them, but the discretization remains still the same.

The variety of tests done show that the higher order multipoint MFD method may have a potentially wide range of applications. The solution of almost all problems, including nonlinear ones may be obtained in this way.

This research is focused on using the meshless finite difference method, and particularly, the higher order multipoint meshless FDM [6,7] for analysis of nonlinear problems.

The paper is organized as follows. In Section 2 the multipoint MFDM solution approach and particularly the general formulation of the method is briefly discussed. Additionally the alternative way to calculate the higher order derivatives is presented in Section 2.4. In Section 3 the multipoint and MFDM approaches for the numerical analysis of nonlinear problem are outlined. The numerical examples, that demonstrate selected results of the MFDM application to the nonlinear problem are presented in Section 4. Section 5 outlines some details of the error analysis. Finally, the paper ends with selected concluding remarks.

2. Higher order multipoint extension of the meshless FDM

2.1. Problem formulation

The higher order multipoint meshless finite difference method, as well as the standard MFDM may be used for analysis of b.v. problems posed in the strong (local), various weak (global), or mixed (local–global) problem formulations. Any formulation which involves an unknown function and its derivatives may be considered here.

The strong formulation, natural for the classical (regular meshes) FDM, is given as a set of differential equations with appropriate boundary conditions

$$\begin{cases} Lu = f, & \text{for} \quad P \in \Omega \\ Gu = g, & \text{for} \quad P \in \partial\Omega \end{cases} \tag{1}$$

where L, G are differential operators and $u = u(P)$.

The weak formulations may be posed either in the form of a functional minimization, or more general as variational principle in the domain Ω

$$b(u,\ v) = l(v), \qquad \forall\ v \in V, \tag{2}$$

where l is a linear form which depends on the test function v, b is a bilinear functional dependent on v and solution u of the considered b.v. problem, and V is the space of test functions.

The trial u (an approximate solution of the problem) and test v functions may be different from each other. Assuming the trial function locally defined on each subdomain Ω_i within the domain Ω, the local–global formulation of the Petrov–Galerkin type may be obtained. Several meshless local Petrov–Galerkin (MLPG) formulations were developed [9] using various types of the test function. In particular, in the MLPG5 formulation [9,10], the test function v may be satisfied locally in each subdomain Ω_i as the Heaviside type test function

$$v = \begin{cases} 1 & \text{in} \quad \Omega_i \\ 0 & \text{outside} \quad \Omega_i \end{cases}.$$

Hence, in the whole domain Ω, inside each subdomain Ω_i, any derivative of test function v is equal to zero. Therefore, the relevant terms in the functional $b(u,\ v)$ and in $l(v)$ vanish, reducing in this way the amount of involved calculations. Therefore, the MLPG5 local–global formulation may be computationally more efficient than the other ones.

2.2. The idea of the multipoint approach

Let us consider the strong (local) formulation of the b. v. problem given in the domain Ω for the n-th order PDE with appropriate boundary conditions (1) or an equivalent weak (global) one formulated as the variational principle (2).

The multipoint method, as well as the MFDM is based on the local MWLS approximation constructed on the node stencil called the MFD star (Fig. 2). The finite difference operator Lu is generated at this stencil.