Engineering Analysis with Boundary Elements 90 (2018) 1–16

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Application of the differential quadrature finite element method to free vibration of elastically restrained plate with irregular geometries

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ARTICLE INFO

Keywords:
DQFEM
Elastic restraints
Irregular domain
Nodes collocations methods

ABSTRACT

The virtual spring technique is firstly introduced into the differential quadrature finite element method (DQFEM) to simulate the practical elastic restraints. The imposing procedures of the boundary conditions are simplified so that a certain kind of restraints can be easily achieved by merely setting different stiffness of the springs. The mapping technique is used to apply the DQFEM to irregular domain. The effects of different nodes collocation methods on the mapping results and vibration results are also discussed, through which one can conclude that the nodes distribution methods affect the accuracy of the mapping technique and the computing time. Especially, the uniformly distributed nodes are not the best selection for mapping process. The Guass Lobatto quadrature nodes are the good choice to obtain the better results in a relatively short time. Several numerical examples are carried out to demonstrate the validity and accuracy of the present solution by comparing with the results obtained by other researchers.

1. Introduction

The finite element method (FEM) and finite difference method (FDM) have been widely used in many engineering problems. In these methods, the functions of an element are usually approximated by the low-order polynomials. However, in order to obtain the high accuracy of the results, the number of elements will increase rapidly and then it will consume more computing resource. Another alternative algorithm also called as the differential quadrature method (DQM) is proposed by Bellman et al. [1,2]. The greatest advantage of this algorithm is merely using a little of grid points to achieve high accuracy. However, there are some drawbacks in determining the weighting coefficients [3] in the original DQM. Many researchers have concerned the application of the multiple boundary conditions in DQM [4–6]. However, up to now there have been no feasible, general and simple techniques [7] to deal with the discontinuity problem when applying the DQM to the vibration analysis for structures with multiple boundary conditions. To solve the above problem, a novel method named as the DQEM which draws lessons from the discrete element idea of the finite element method is formulated [8–10]. However, from the existing literature, we can know that the solving process of DQEM is very cumbersome and not conducive to study the complex structure forms. Recently, a differential quadrature finite element method (DQFEM) has been proposed by Yufeng Xing [11]. In DQFEM, the Hamilton's principle is used to derive the equation of motion of a plate for the free vibration analysis, which is similar to the FEM. The discretization is operated on the partial derivative in the strain energy and kinetic energy, which is different from the operation in most of the literatures, where the discretization is operated directly on the differential equations [12–15]. Also, in DQFEM the boundary conditions are imposed similarly as that in FEM while in other literatures the discretized expressions of classical boundary conditions are imposed to modify the coefficient weighting matrix, which makes it complicated to apply the multiple boundary conditions to the structures.

The DQFEM has been used to solve many vibration problems with irregular geometries [16]. However, most of the researches only considered the classical boundary conditions of the structures while few works on the vibration characteristic of structures with elastic restraints can be found. The elastic restraints are simulated by employing translational and rotational springs linking the structure and the ground [17]. By setting different spring stiffness to achieve desired boundary conditions including several ideal classical boundary conditions. Many works have been done on the influence of spring stiffness on the vibration characteristic of the structure [17–23]. However, according to the available literatures, most of the structures are of regular geometries, such as rectangular [22,23], circle [18], sector [18], etc. The main purpose of the authors is to introduce the concept of elastic restraints into the DQFEM to expand the application of the method to the analysis of vibration problems with elastic restraints as well as irregular domains.

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https://doi.org/10.1016/j.enganabound.2018.02.005
Received 7 April 2017; Received in revised form 13 February 2018; Accepted 13 February 2018
Available online 23 February 2018
0955-7997/

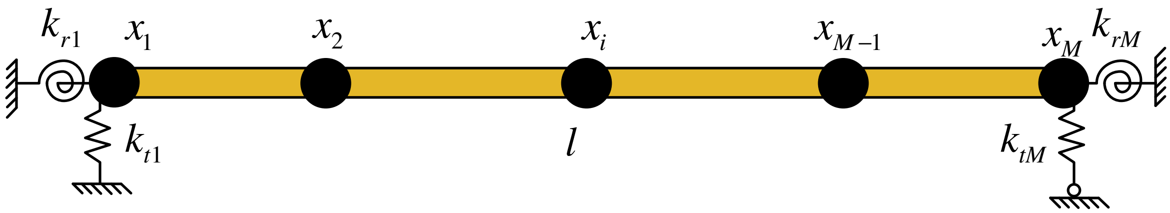

Fig. 1. The beam with elastic restraints on both ends.

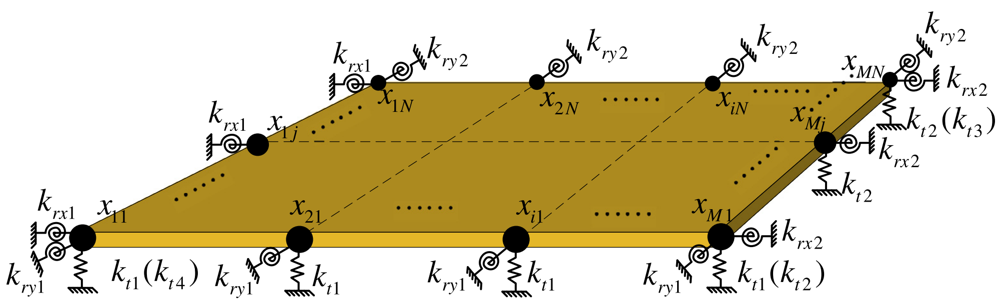

Fig. 2. Rectangular plate with elastic restraints at four sides.

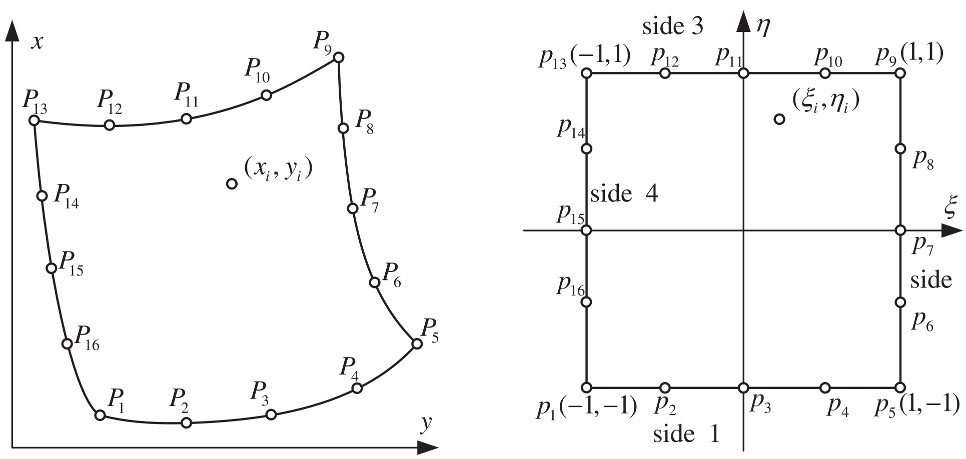

Fig. 3. Cartesian coordinate system and natural coordinate system.

Since the original DQM cannot be applied for the irregular domain directly, a mapping technique is introduced in order to expand the utilization of DQM into irregular domain. Based on the above technique, Fantuzzi, N [14,15] used the quadratic serendipity element to transform the coordinates in Cartesian coordinate system into natural coordinate system. Liu Bo [7] presented the results obtained from cubic and quartic serendipity elements. However, the studies of higher-order serendipity element and the distributions of the element nodes on the accuracy of the mapping process are infrequent.

Based on the lack of researches mentioned above, the main contents of this paper are shown as follows: Firstly, introduce the basic rules of DQ and elaborate the specific principles of DQFEM. Then, a beam structure with elastic restraints is given to illustrate how the virtual translational and rotational springs are applied into equations of motion. Next, an example of a rectangular plate with elastic restraints is given to illustrate the procedures of applying the elastic restraints to a two-dimensional problem. Finally, the vibration of a plate with curved side is presented. The influence of higher-order serendipity element and the distributions of element node on the accuracy and of the mapping technique and the natural frequencies of the plate are discussed. Some numerical results are presented and compared with the literatures available, which proves the accuracy of the present work.

2. Theory and formulations

2.1. The DQ rules

Considering a one-dimensional function $f(x)$ that is derivable in the interval $[a, b]$, and according to the DQ rule, the first-order derivative of function $f(x)$ can be written as

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \sum_{j=1}^{N} W_j(x) f(x_j) \tag{1}$$

where $W_j(x)$ is interpolation basis function, which is N-1 order when polynomials are used in the interpolation. x_j is coordinates of jth grid point among N inequality points defined in $a = x_1 < x_2 < \ldots < x_N = b$. Let $A_{ij} = W_j(x_i)$ and $f_j = f(x_j)$, then the first-order derivative of function $f(x)$ at ith grid point x_i can be written as

$$\left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_i = \sum_{j=1}^{N} A_{ij}^{(1)} f_j \tag{2}$$

where $A_{ij}^{(1)}$ is the first-order weighting coefficient, thus $[A_{ij}^{(1)}]$ is the first-order weighting coefficient matrix. The sth-order derivative of the