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An extended polygon scaled boundary finite element method for the nonlinear dynamic analysis of saturated soil

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ABSTRACT

In this paper, the polygon scaled boundary finite element method is extended to analyze saturated soil based on the generalized Biot's dynamic consolidation theory. The displacement shape functions of the polygon element are obtained by elastic static theory while the pore pressure shape functions are constructed from steady-state seepage theory. A scaled boundary polygon equations for saturated soil is established by applying Galerkin method. Two sets of Gauss points are adopted, including Gauss points of line utilized to compute the shape functions and Gauss points of area employed to realize material nonlinearity. In order to verify and assess the reliability and accuracy of the presented method, a saturated elastic half space subjected to a uniform cyclic dynamic loading is simulated and the results are compared with the analytical solution. Moreover, a liquefaction analysis of a breakwater built on saturated sand soil with generalized plastic model is subsequently carried out. The results correspond well with those calculated by finite element method (FEM), which indicates the significant capability of the current method in solving nonlinear problems. The proposed method processes extraordinary mesh flexibility and fast reconstruction, which will make it a promising tool in liquefaction analysis.

1. Introduction

As saturated soil is pervasive in the environment, its response to dynamic loading, such as earthquake, is of particular importance in a vast number of practical engineering problems. Saturated soil is a kind of saturated porous medium, which is a two-phase medium composed of a solid phase and a fluid phase. The development of pore fluid pressure during dynamic loading may significantly affect the dynamic response of a structure built on saturated soil, such as the liquefaction phenomenon in saturated sand soil. Considering the coupling interaction between solid skeletons and pore fluid, Biot [1] firstly proposed a set of governing equations which is accurate and reliable in modeling dynamic behavior of saturated porous media. Biot's theory has been applied to various problems in acoustics, geotechnical and other fields up to date. However, since the coupled partial differential equations are difficult to solve exactly, the analytical solutions are unavailable for all but the simplest problems [2–5]. Therefore, applying numerical methods, such as finite element method (FEM) and boundary element method (BEM) [6–9], is a feasible way to obtain solutions to complex problems.

The scaled boundary finite element method (SBFEM) is a semi-analytical method proposed by Song and Wolf [10–12], which combines the advantages of FEM and BEM. In this method, the discretization is only conducted in the circumferential direction and there is no need to introduce a fundamental solution. In addition, it satisfies the singular problem automatically. Compared with conventional FEM, SBFEM provides a high precision solution with a rapid convergence rate and significantly reduces the degree of freedom in computational model [13]. These advantages make it a powerful numerical method. Since it was proposed, the method has been applied to many problems in engineering practice such as unbounded media [14], electrostatic fields [15], crack propagation [16], magneto-electro-elastic plate [17], fluid–structure interaction [18], layered soil [19] and heat conduction [20]. In unbound media problem, the boundary condition at infinity can be satisfied exactly. In crack propagation problem, the singularity can be handled without additional effort such as local mesh refinement. In fluid–structure interaction problem, since the discretization is performed only at the boundary, the number of degrees of freedom in the computational model is reduced to a large extent.

The polygon scaled boundary finite element method (PSBFEM) was recently established based on SBFEM which provides a great flexibility in modeling complex geometries and is an essential complement to the original SBFEM. Compare with FEM, it exhibits high precision with more rapid convergence rate and an advantage of solving singularity problem which is inherited from the SBFEM [21–23]. Many researchers have applied this method to their professional fields. Ooi et al. [24,25] and Dai et al. [26] applied this theory to model the crack propagation problems.

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Bao et al. [27] conducted a fracture analysis of a gravity dam under seismic loading, and Chiong et al. [28] performed a fracture analysis of functionally graded materials. Luo et al. [29] simulated a grain breakage problem using SBFEM in combination with discrete element method (DEM) where a scaled boundary polygon element is used to model an individual grain.

SBFEM is a versatile and efficient numerical method that has been widely used in various engineering and research fields as mentioned above. Nevertheless, most of the application fields are concentrated on single-phase medium and there are almost no reports about applying this method into saturated soil. In this paper, the PSBFEM is extended to the nonlinear dynamic analysis of saturated soil based on the generalized Biot's dynamic consolidation equations derived by Zienkiewicz et al. [30]. The displacement shape functions of polygon elements are constructed from the SBFEM equation of the elastic static problem while the pore pressure shape functions of polygon elements are constructed from the SBFEM equation of steady-state seepage problem. And then Galerkin method and Green formula are applied to the generalized Biot's dynamic consolidation equations, resulting in a spatially discretized scaled boundary polygon equations for dynamic analysis of saturated soil. By introducing Gauss points of area into each polygon, the material nonlinear can be considered in this equation set. The proposed method that can be discretized with arbitrary polygon and quadtree mesh is flexible in modeling complex geometry and fast and automatic in mesh generation, which makes it a competitive numerical tool in practical engineering.

The rest of this paper is organized as follows. In Section 2, the construction of scaled boundary polygon displacement shape functions and pore pressure shape functions is illustrated in details. In addition, the strain–displacement transformation matrix and the transformation matrix between pore pressure gradient and nodal pore pressures are described. The derivation of scaled boundary polygon equations for dynamic analysis of saturated soil is described in Section 3. Section 4 introduces the development platform of the proposed method. Two numerical examples are simulated to validate the reliability and accuracy of the presented method in Section 5, followed by the conclusions in Section 6.

2. Scaled boundary polygon shape functions

2.1. Coordinates transformation

An arbitrary domain can be discretized with a mesh of arbitrary n-sided polygons (where n is larger than 2). An arbitrary polygon can be treated as a SBFEM subdomain as long as the so-called scaling center $\boldsymbol{O}$ is chosen in a zone, from which the total boundary is visible. The numerical results of the domain are obtained after solving each subdomain with the SBFEM. A typical polygon modeled using the SBFEM is shown in Fig. 2.1. A scaling center is defined at the geometric center of the polygon. Each edge of the polygon is discretized using a one-dimensional line element with a local coordinate that varies from −1 to 1, and a radial coordinate is defined that varies from 0 at the scaling center to 1 at the boundary. The Cartesian coordinates of a point on a line element with M nodes can be expressed by the scaled boundary coordinates as

$$x(\eta) = \boldsymbol{N}(\eta)\boldsymbol{x}_b \tag{2.1}$$

$$y(\eta) = \boldsymbol{N}(\eta)\boldsymbol{y}_b \tag{2.2}$$

with

$$\boldsymbol{N}(\eta) = \left[N_1(\eta),\ N_2(\eta),\ \cdots,\ N_M(\eta)\right] \tag{2.3}$$

where $\boldsymbol{x}_b$ and $\boldsymbol{y}_b$ are the vectors of nodal coordinates of a boundary line element, $x(\eta)$ and $y(\eta)$ are the coordinates along the line element, and $\boldsymbol{N}(\eta)$ is the shape function vector of the line element. M can be any number larger than or equal to 2 which increases with the order of the shape functions of the line element. In this paper, first order Lagrange shape

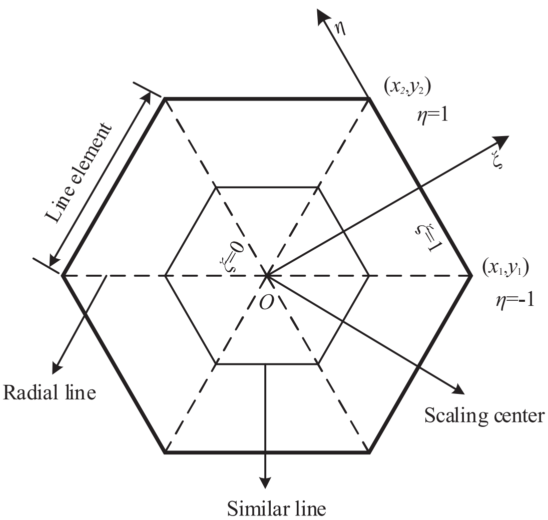

Fig. 2.1. Polygon representation for SBFEM.

functions are adopted where M equals 2. However, it is convenient to use high order shape functions without additional effort in mesh generation. The whole domain of the polygon can be described by scaling the boundaries according to the radial coordinate ξ. The Cartesian coordinates of a point within the domain with the origin at the scaling center can be given by the scaling boundary transformation equations as

$$x(\xi,\eta) = \xi\boldsymbol{N}(\eta)\boldsymbol{x}_b \tag{2.4}$$

$$y(\xi,\eta) = \xi\boldsymbol{N}(\eta)\boldsymbol{y}_b \tag{2.5}$$

2.2. The scaled boundary polygon displacement shape functions

In this study, the displacement shape functions are constructed from an elastic static equilibrium problem. Within a subdomain covered by a line element on a polygon boundary, the displacement of a point can be given as below using scaled boundary coordinates

$$\boldsymbol{u}(\xi,\eta) = \boldsymbol{N}_u(\eta)\boldsymbol{u}(\xi) \tag{2.6}$$

where $\boldsymbol{u}(\xi)$ are the radial displacement functions along a line connecting the scaling center and a node on the boundary, and $\boldsymbol{N}_u(\eta)$ is the displacement shape function matrix along the circumferential direction with the following form

$$\boldsymbol{N}_u(\eta) = \begin{bmatrix} N_1(\eta) & 0 & \cdots & 0 & N_M(\eta) & 0 \\ 0 & N_1(\eta) & 0 & \cdots & 0 & N_M(\eta) \end{bmatrix} \tag{2.7}$$

The radial displacement functions $\boldsymbol{u}(\xi)$ are the solution of the SBFEM governing equations in displacement

$$\boldsymbol{E}_0\xi^2\boldsymbol{u}(\xi)_{,\xi\xi} + \left(\boldsymbol{E}_0 - \boldsymbol{E}_1 + \boldsymbol{E}_1^T\right)\xi\boldsymbol{u}(\xi)_{,\xi} - \boldsymbol{E}_2\boldsymbol{u}(\xi) + \boldsymbol{F}(\xi) = \boldsymbol{0} \tag{2.8}$$

where $\boldsymbol{E}_i (i = 0, 1, 2)$ are coefficient matrices depending only on the geometry and material properties of the subdomain and $\boldsymbol{F}(\xi)$ is a load vector including contributions from side-face traction, body force and thermal loads. When $\boldsymbol{F}(\xi) = 0$, the second order nonhomogeneous ordinary differential equations in Eq. (2.8) can be transformed into first order homogeneous ordinary differential equations via introducing a new vector $\boldsymbol{X}(\xi)$ as below

$$\boldsymbol{X}(\xi) = \begin{Bmatrix} \boldsymbol{u}(\xi) \\ \boldsymbol{q}(\xi) \end{Bmatrix} \tag{2.9}$$

$$\xi\boldsymbol{X}(\xi)_{,\xi} = -\boldsymbol{Z}X(\xi) \tag{2.10}$$

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