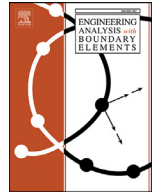




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## Application of the generalized finite difference method to three-dimensional transient electromagnetic problems

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## ABSTRACT

We apply the generalized finite difference method (GFDM), a relatively new domain-type meshless method, for the numerical solution of three-dimensional (3D) transient electromagnetic problems. The method combines Taylor series expansions and the weighted moving least-squares method. The main idea here is to inherit the high-accuracy advantage of the former and the stability and meshless attributes of the latter. This makes the method particularly attractive for problems defined in 3D complex geometries. Three benchmark 3D problems governed by the Maxwell's equations with both smooth and piecewise smooth geometries have been analyzed. The convergence, accuracy and stability of the method with respect to increasing the number of scattered nodes inside the domain are studied.

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## 1. Introduction

The finite element (FEM), finite difference (FDM) and boundary element methods (BEM) have long been dominant numerical methods in the simulation of electromagnetic problems. However, the mesh generation of these methods for problems involving, for example, mesh distortion, large deformation and moving boundary, remains challenging [1–4]. In order to alleviate some of these difficulties, the past few decades have witnessed considerable effort to eliminate the need for meshing. This led to the development of meshless or meshfree methods which require neither domain nor boundary meshing. The research of meshless methods nowadays is highly valued and has become one of the hotspots in science and engineering computation [5–11].

Generally speaking, the meshless methods can be divided into the boundary-type [12,13] and domain-type [14,15] techniques according to whether the interpolation basis functions satisfy the governing equation. The boundary-type meshless methods still inherit the advantage of the BEM to reduce the dimensionality of the problem and be able to solve challenging problems effectively. In recent years, boundary-type meshless methods have rapidly developed and the methods developed so far include, but are not limited to, the method of fundamental solutions (MFS) [16], the boundary knot method (BKM) [17], the boundary element-free method (BEFM) [18], the singular boundary method (SBM) [12,19], and the regularized meshless method (RMM)

[20]. For an overview of the state of the art, we refer the reader to Refs. [12,21], as well as the references therein. At present, the more commonly used domain-type meshless methods include the element-free Galerkin method (EFGM) [22], diffuse element methods (DEM) [23], the smoothed particle hydrodynamics method (SPGM) [24], and the generalized finite difference method (GFDM) [25,26] etc. Among these methods, the GFDM is a relatively new domain-type meshless method for the numerical solution of boundary value problems governed by certain partial differential equations. In the GFDM which is based on the Taylor series expansion and weighted least squares fitting technique, the partial derivatives of the unknown variables at each nodal point are approximated by a linear combination of the adjacent nodal function values. In addition, the coefficient matrices generated by the GFDM are sparse, which can be solved efficiently by using various sparse matrix solvers. At present, the method has been well developed and has been successfully used in solving various scientific and engineering problems. A self-adaptive GFDM was proposed by Benito et al. [25], which automatically distributes the collocation points according to the required accuracy. Urena et al. [27] extended the GFDM to solve third- and fourth-order partial differential equations. Gavete et al. [28] summarized the advantages and disadvantages of the method, which can be viewed as a good guide for using the GFDM. In a more recent study, Gu et al. applied the method for the numerical solution of inverse heat source problems [29].

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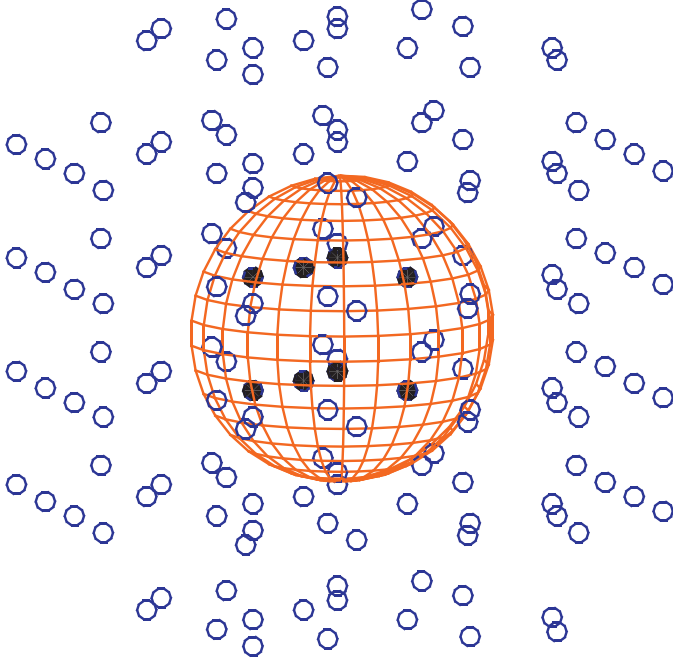
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**Table 1**  
Convergence study of GFDM as the number of nodes increase.

Number of nodes	Relative error of $A_x$	Relative error of $A_y$	Relative error of $A_z$	Relative error of $\phi$
1000	2.78E-5	1.43E-5	1.82E-5	4.29E-5
1331	2.50E-5	3.40E-5	4.56E-6	1.01E-5
1728	8.87E-6	4.49E-6	4.60E-6	8.68E-6
3375	5.80E-6	3.80E-6	2.10E-6	4.23E-6



**Fig. 1.** Diagram of the GFDM.

Inspired by the pioneering works described above, this paper makes the first attempt to apply the GFDM for the numerical solution of 3D transient electromagnetic problems governed by Maxwell's equations. The implicit Euler method is employed to discretize the time-domain part, while the remaining coupled partial differential equations in space are solved by using the GFDM proposed in this paper. The MATLAB computer program is developed for general 3D transient problems and validated using the analytical solution of several benchmark problems. The developed GFDM approach can provide not only a robust numerical tool for the solution of Maxwell's equations, but also the basis for further investigations of dynamic problems, such as elastodynamic analysis, crack and/or wave propagation.

The paper is organized as follows: The mathematical formulation for general 3D transient electromagnetic problems is introduced in Section 2. The GFDM formulation and its numerical implementation are presented in Section 3. Numerical results are presented in Section 4 for three benchmark test problems in both smooth and piecewise smooth geometries. Finally, some conclusions and remarks are provided in Section 5.

## 2. Mathematical model of 3D transient electromagnetic field problem

The fundamental equations describing the behavior of electric and magnetic fields are the well-known Maxwell's equations, which can be expressed as:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (4)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  present the electric and magnetic fields, respectively,  $\mathbf{D}$  and  $\mathbf{B}$  stand for the electric and magnetic induction, respectively,  $\mathbf{J}$  is the free current density, and  $\rho$  is the free charge density. The following constitutive relations are assumed:

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{J} = \sigma \mathbf{E}, \quad (5)$$

where  $\epsilon$  and  $\mu$  are the dielectric and magnetic permeability coefficients, respectively, and  $\sigma$  is the electric conductivity.

Eq. (4) states that the magnetic field  $\mathbf{B}$  is solenoidal, so it can be expressed as the curl of a vector potential. Then, the magnetic vector potential  $\mathbf{A}$  is introduced to satisfy the relation  $\nabla \times \mathbf{A} = \mathbf{B}$ .

Substituting  $\nabla \times \mathbf{A}$  for  $\mathbf{B}$  into Eq. (2), we obtain

$$\nabla \times \left( \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0. \quad (6)$$

The curl will be identically zero if the vector is equal to the gradient of a scalar potential, thus, an electric scalar potential  $\phi$  is introduced, so that the magnetic field  $\mathbf{B}$  and electric field  $\mathbf{E}$  can be represented by the following equations

$$\begin{cases} \mathbf{B} = \nabla \times \mathbf{A}, \\ \mathbf{E} = -\left( \frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right). \end{cases} \quad (7)$$

Eqs. (1)–(4) can now be expressed as

$$\begin{cases} \frac{1}{\mu_0} \nabla \times \nabla \times \mathbf{A} + \sigma \left( \frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) = \frac{\partial \mathbf{D}}{\partial t}, \\ \nabla \times \sigma \left( \frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) = -\frac{\rho}{\epsilon}. \end{cases} \quad (8)$$

subject to boundary/initial conditions

$$\begin{cases} \mathbf{A} = \bar{\mathbf{A}}, \quad \phi = \bar{\phi}, & (\text{Dirichlet boundary conditions}) \\ \mathbf{e}_n \cdot \nabla \mathbf{A} = \bar{q}_A, \quad \mathbf{e}_n \cdot \nabla \phi = \bar{q}_\phi, & (\text{Neumann boundary conditions}) \\ \mathbf{A}|_{t=t_0} = \mathbf{A}_0 & (\text{Initial conditions}). \end{cases} \quad (9)$$

where  $t_0$  denotes the initial time. Eq. (8) can be written in component form as

$$\begin{cases} \frac{1}{\mu_0} \left( \frac{\partial^2 A_y}{\partial x \partial y} - \frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial^2 A_z}{\partial x \partial z} \right) + \sigma \frac{\partial A_x}{\partial t} + \frac{\partial \phi}{\partial x} = \tilde{f}_{1x}, \\ \frac{1}{\mu_0} \left( \frac{\partial^2 A_z}{\partial y \partial z} - \frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_x}{\partial x \partial y} \right) + \sigma \frac{\partial A_y}{\partial t} + \frac{\partial \phi}{\partial y} = \tilde{f}_{1y}, \\ \frac{1}{\mu_0} \left( \frac{\partial^2 A_x}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_y}{\partial y \partial z} \right) + \sigma \frac{\partial A_z}{\partial t} + \frac{\partial \phi}{\partial z} = \tilde{f}_{1z}, \\ \frac{\partial}{\partial t} \left[ \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right] + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \tilde{f}_2. \end{cases} \quad (10)$$

The transient electromagnetic field problem can be summarized by equations (8) or (10). In particular, when the induced electric field is much smaller than the Coulomb electric field, the  $\partial \mathbf{B} / \partial t$  term can be ignored (electroquasistatic case [30]); when the displacement current density is far less than the conduction current density, the  $\partial \mathbf{D} / \partial t$  term can be ignored (magnetoquasistatic case [31]). Similarly, the computation of the Eddy current field and the electric field are both quasi-steady electromagnetic field problems.

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