Engineering Analysis with Boundary Elements 90 (2018) 47–62

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Quick and robust meshless analysis of cracked body with coupled generalized hyperbolic thermo-elasticity formulation

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ARTICLE INFO

Keywords:
Meshless method
Coupled thermo-elasticity
Hyperbolic non-Fourier theory
Shock, Lord–Shulman
Transient SIF

ABSTRACT

In this article, the MLPG method is applied to the generalized linear coupled thermoelectricity equations. Lord–Shulman modification with a relaxation time parameter is used in the hyperbolic heat conduction equations. A new linear test function which is zero on the boundaries of local test domains is introduced. The test function and its partial derivatives are determined by an exponential RBF approximation method. The approximation of test function and main variables are similar. For the construction of shape functions, neighbors of every point are determined based on the definition of the closest adjacent point pattern. Consequently, test function space becomes independent of trial function space. Direct interpolation method and penalty parameter are used to impose essential boundary conditions. The selection of appropriate parameters are demonstrated in two numerical examples. The small number of used points is the advantage of this method over the FEM that is shown in several examples. The accuracy of results is compared between the meshless method and different analytical and FE solutions. The effect of the relaxation time on SIF under thermal shock is discussed in a separate example. The comparison of meshless results with various examples shows that employed method is accurate and reliable.

1. Introduction

Although analytical methods provide closed-form solutions for engineering problems, there is much computational and mathematical complexity in such solutions. Thus in order to reach more efficient and general solutions, numerical methods like finite element method [1] or boundary element method [2,3] are required. Until now, the extended finite element method [1,2,4,5] has been widely used as a powerful numerical method of diverse problems like crack growth [5], stationary crack analysis under dynamic or static loads [1,4] and transient analysis of cracked magnetoelectroelastic solids under coupled electro-magneto-mechanical loading [6]. The conventional form of FE methods encounters some difficulties in problems in which remeshing is required, large deformations with high distortions of the mesh exist or crack propagates. Furthermore, the accuracy of the numerical solution depends on the meshing quality. To overcome these problems, the meshless methods appeared since the late of 90s, and now they are developing in different branches of science, especially mechanical engineering. However, the use of these methods may be challenging due to the mathematical complexities and being time-consuming compared with the finite element method. These methods are free of any mesh and only need some scattered points in the configurations [7–10]. In past years, many researchers applied meshless methods to different engineering problems. For example, Tanaka et al. [11,12] used a novel meshfree discretization technique in terms of the reproducing kernel particle method for evaluating mixed-mode SIFs of cracked plates. They employed enriched basis functions for approximation and Voronoi meshing for numerical integration. Nguyen et al. [13] presented extended meshfree Galerkin method based on local partition of unity for modeling of crack growth. They used radial point interpolation method (RPIM) with enriched basis functions for generating the TPS shape functions. Bui et al. [14] used meshfree moving Kriging interpolation method to analyze the natural frequencies of laminated composite plates. Bui et al. [15] numerically analyzed the transient dynamic SIFs of cracked FGMs by extended meshfree methods and extended moving Kriging shape functions. They calculated SIFs by interaction integral method and compared their results with analytical method, XFEM and boundary element ones. Sadamoto et al. [16] studied the buckling of cylindrical shells and calculated the critical buckling loads and their mode shapes by the meshfree reproducing kernel method. Hosseini [17] used a meshless method based on the generalized finite difference method for generalized coupled thermoelasticity analysis based on the Green–Naghdi (GN) theory [18]. Different classification can be considered for meshless methods based on the formulation (global or local), weak form or strong form of the governing equations, type of test functions, approximation method for construction of shape functions and method of enforcing essential boundary

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https://doi.org/10.1016/j.enganabound.2018.02.006
Received 20 November 2017; Received in revised form 8 January 2018; Accepted 13 February 2018
0955-7997/

conditions. The meshless methods which are based on global formulation are not completely meshfree in fact. This means that to calculate the integrals in the weak form a background mesh is required like [11]. Of course, Racz and Bui [19] proposed an adaptive numerical integration method based on mapping techniques for solving domain integrations. Their method maps complex domains to simpler ones applicable to both global and local weak forms. This method is useful but needs some additional calculations of mapping process and a CAD program. Additionally, more calculation time is required. Local meshfree methods such as Petrov-Galerkin method do not need background mesh because of their local nature, nevertheless, the idea of using background mesh is feasible.

In this paper, the weak form meshfree method of local Petrov–Galerkin (MLPG) is applied to the generalized (hyperbolic) coupled linear thermoelectricity governing equations based on Lord–Shulman method with one time lag parameter. Some researchers studied this area by methods rather than meshless method. For example, Alibeigloo [20] analytically studied the time-dependent response of sandwich plates with FGM core under thermal shock by using generalized coupled thermoelasticity based on the Lord–Shulman formulation. He used Laplace and Fourier transformations. Heidarpour and Aghdam [21] used differential quadrature method (DQM) to analysis transient response of FGM shells under thermal shock load based on the Lord–Shulman model. Gau and Wang [22] analytically studied the thermal shock fracture of penny-shaped crack based on non-Fourier heat conduction theory by Laplace transform method. Chen and Hu in [23] analytically studied the response of a cracked substrate bonded to a coating using the hyperbolic heat conduction theory and employed Laplace and Fourier transforms. Hosseini et al. used MLPG method in [24] and presented an analytical solution in [25] to study coupled thermoelasticity analysis of FG thick hollow cylinder under thermal shock based on GN theory. Furthermore, thermomechanical and shock loads have been studied in [26,27] by FEM. Liu et al. [26] studied stationary cracks in FG piezoelectric materials (FGPMs) based on the X-FEM under both cooling and heating thermal shocks. They compared FEM results with MLPG ones. Nguyen et al. [27] simulated dynamic and static thermoelastic fracture by extended nodal gradient FE method. In addition, they investigated the simulation of quasi-static crack propagation in complex geometries under thermo-mechanical loading. But, the novelty of this article compared with the mentioned articles is applying MLPG method to the generalized form of the coupled linear thermoelectricity governing equations based on the LS model and considering the second sound effect. The results of numerical investigations of cracked homogenous and FG materials under various time-dependent and time independent thermal and mechanical loads are compared with reference numerical and analytical ones. Thus, the discrete form of the non-Fourier thermoelasticity equations is determined based on the MLPG method and general forms of the mass matrix, damping matrix and stiffness matrix and the force vector are derived. The selection of appropriate parameters of exponential shape function and penalty parameter, numerical integration procedure, the new method of construction of test function based on RBF approximation are explained in details. To evaluate the accuracy of the numerical method, various examples are presented. To calculate the stress intensity factors, the equivalent domain form of J-integral, interaction integral [28–30], are used.

Table 1
RBF definition.

Name	Expression	Parameter
Gaussian (EXP)	$R_i(\boldsymbol{x}) = \exp(-c^2 r_i^2)$	$c = \sqrt{\alpha}/(\sqrt{A}/\sqrt{n} - 1)$

This work is structured as follows. After the introduction, the shape function approximation method and the appropriate parameter selection are explained in the second section. Afterwards, the generation of shape functions and the method of neighbor points allocation is presented in the third section. It is required to enforce discontinuities arising from the crack on the shape functions. This is discussed in the fourth section. Next, in the fifth section, the discretization procedure of the governing equations is presented. Afterwards, the enforcement of the essential boundary conditions and the procedure of numerical integration are discussed in the sixth and seventh sections, respectively. The calculation of the SIFs by interaction integral method and necessary details about it are given in the eighth and ninth sections. Numerical results are presented in the tenth section and they are compared with different analytical and FEM results. In this section, the effect of the relaxation time parameter on the SIFs is investigated under thermal shock loads. Finally, some conclusions are given in the eleventh section.

2. Interpolation/Approximation

Meshfree methods are based on approximating variables (for example, u) in the scattered points by their adjacent points without any mesh. u may be each of displacement components in three directions of an orthogonal coordinate system, temperature or any other engineering variable. For the approximation of such these variables at an arbitrary point $\boldsymbol{x}$, Eq. (1) is used:

$$u = \boldsymbol{\phi}\hat{\boldsymbol{u}} \tag{1}$$

In which $\boldsymbol{\phi}_{(1\times n)}$ is the row vector of shape functions and $\hat{\boldsymbol{u}}_{(n\times 1)}$ is the unknown column vector of nodal values of the neighbor points. n is the number of neighbor points within the support domain of point x. The procedure of shape function construction includes different types in meshless methods. Two of the most common methods are the moving least squares and the radial basis functions (RBFs). In this paper, we use the radial basis functions. To get into Eq. (1), first the interpolation is defined as below [31]:

$$u(\boldsymbol{x}) = \sum_{i=1}^{n} R_i(\boldsymbol{x})a_i + \sum_{j=1}^{m} p_j(\boldsymbol{x})b_j = \boldsymbol{R}^T(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{p}^T(\boldsymbol{x})\boldsymbol{b} \tag{2}$$

$\boldsymbol{R}$ is the radial basis function vector; $\boldsymbol{P}$ is the monomials' vector; $\boldsymbol{a}$ and $\boldsymbol{b}$ are unknown vectors. m is the number of polynomial basis functions. If $m=0$ is selected, it is called classical RBFs and if $\boldsymbol{P}$ is not empty, it is called the enriched RBF. $\boldsymbol{R}$ is expressed as below:

$$R_i(\boldsymbol{x}) = f(r_i) = f\left(\sqrt{(x-x_i)^2 + (y-y_i)^2}\right) \tag{3}$$

This means that each radial function is described as a function of the radial distance between the selected point and its adjacent points. In this paper, exponential RBF is used as described in Table 1 [31]:

In the above relations, $r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$ and α is the shape parameter. A is local interpolation area containing adjacent points. In this study, reasonable results have been observed for values of α between 2 and 3. For all numerical examples, $\alpha = 2.5$ has been used. Other RBFs such as TPS, MQ, and CSRBF are also available for interpolation. For more details, references [31] are recommended.

In two dimensional space, $\boldsymbol{P}$ is expressed based on the value of m as follows [32]:

$$\boldsymbol{p} = \{1 \quad x \quad y\}^T \qquad m = 3,\ \text{first order}$$

$$\boldsymbol{p} = \{1 \quad x \quad y \quad x^2 \quad xy \quad y^2\}^T \qquad m = 6,\ \text{second order}$$

In 2D problems, to stabilize RBFs and approximate the possible linear space appropriately, using first order monomials with $m=3$ has been recommended. Therefore, in this paper $m=3$ is taken.

To find the unknowns $\boldsymbol{a}$ and $\boldsymbol{b}$ and to determine $\boldsymbol{\phi}$, the interpolation has to be applied to all n points within the support domain of $\boldsymbol{x}$. In this order, the following relations are obtained [31]:

$$\hat{\boldsymbol{u}} = \boldsymbol{R}_0\boldsymbol{a} + \boldsymbol{p}_m\boldsymbol{b} \tag{4}$$