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An iterative approach for analyzing cracks in two-dimensional piezoelectric media with exact boundary conditions

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ABSTRACT

An iterative approach is proposed for analyzing cracks in two-dimensional piezoelectric media based on the sub-region boundary element formulation with exact boundary conditions on the crack face. In this approach, the opening crack cavity is treated as one of the sub-regions governed by the Poisson equation of electric potential. The real crack opening, the crack cavity region, and the field in the cavity region are determined for the geometrically nonlinear problem. The widely used approximate crack models corresponding to three approximate boundary conditions are evaluated based on the proposed approach. The proposed approach is implemented efficiently to analyze problems with very thin domains, such as the crack opening cavity, and employing the intelligent adaptive algorithms in the Mathematica software means that nearly singular integrals require no special treatment.

1. Introduction

Piezoelectric materials such as quartz crystal, piezoelectric ceramic transducer (PZT), barium titanate (BT), and ceramics are used in engineering structures because of their attractive features. Research on the fracture of these materials has received significant attention, as reviewed in several papers [1–7]. Cracks open under combined mechanical-electrical loadings, and an electric field can develop in the crack cavity. The electric field in the crack cavity field can in turn significantly affect the deformation, e.g., the crack opening [3]. Therefore, the crack problem in a piezoelectric medium is typically geometrically nonlinear. To simplify the problem, three kinds of approximate electric boundary conditions along the crack face, also called approximate crack models, have been proposed and are widely used. One is the electrically impermeable crack [8], in which the normal electric displacement on the crack face is assumed to be zero. The second is the electrically permeable crack [9], in which the electric potential and the normal electric displacement component on each side of the crack face are assumed to be equal. The third model, which is electrically semi-permeable crack, assumes that the normal electric displacement is proportional to the electric potential jump divided by the opening displacement discontinuity along the crack. As these simplified crack models are all approximate, solutions for the electric quantities based on these models are quite different. In 1998, Zhang et al. [11] presented an analytical solution for a crack in an infinite piezoelectric medium. They used the exact electric boundary condition on the crack face based on the self-consistent approach, and evaluated the effects of the three approximate boundary conditions. It was found that different approximate boundary conditions lead to different solutions. Thus, solutions based on the exact boundary conditions are essential to understanding the fracture behavior of piezoelectric materials.

Analytical solutions are usually difficult to obtain for nonlinear problems with a finite domain and non-uniform loads. Therefore, we have to resort to numerical methods to solve such complex problems. The boundary element method (BEM) is an effective tool to solve crack problems. To overcome the degeneration of the two surfaces of a crack, several modified BEM methods have been proposed. Snyder and Curse [12] applied special Green's functions to line cracks to avoid the discretization of the two sides of the crack. Crouch [13] proposed the displacement discontinuity method (DDM), in which the discretization is only required on one side of the crack face. Blandford et al. [14] developed the sub-region method, in which the cracked domain is divided into several sub-regions separated along the crack faces. Another approach to handle fracture mechanics is the dual boundary element method (DBEM), as presented by Hong and Chen [15] and Portela and Aliabadi [16]. Instead of collocating points on both sides of the crack surface, Pan and Amadei [17] developed the single-domain BEM, in which the relative crack displacements are collocated along only one

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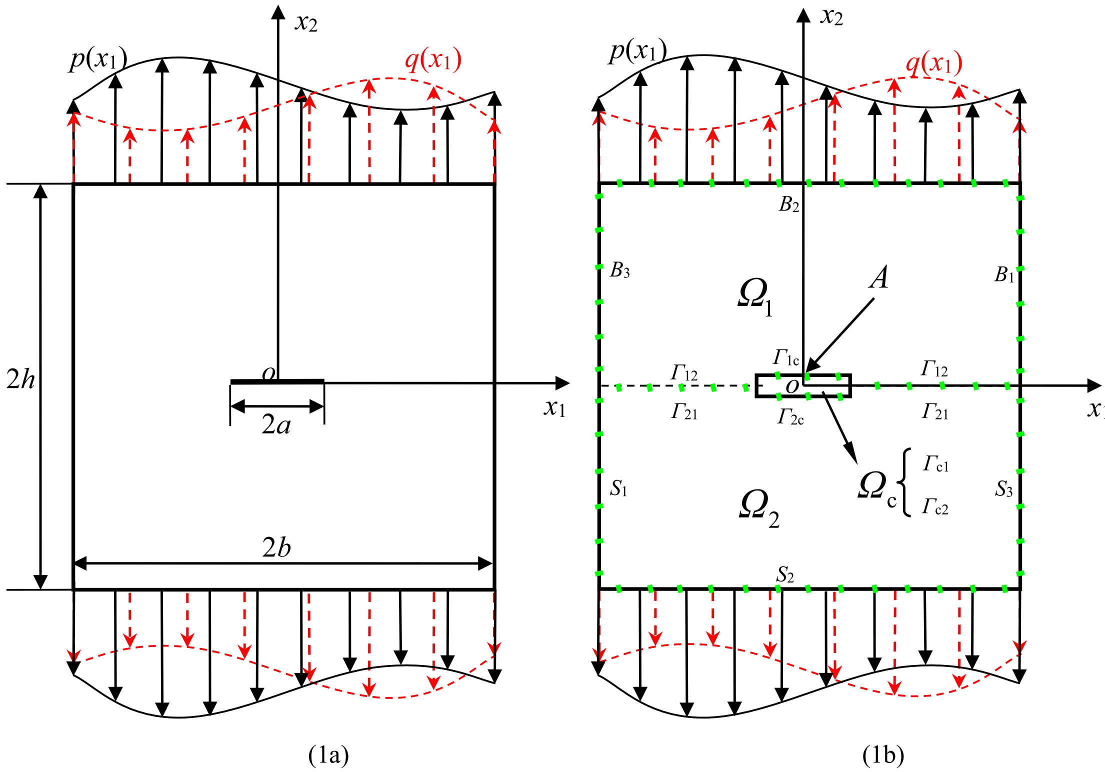

Fig. 1. (a) A central crack in a piezoelectric rectangular plate under mechanical $p(x_1)$ and electric $q(x_1)$ loads over its top and bottom surfaces. (b) The sub-regions of the central crack model and discretization of the involved boundaries.

side of the crack surfaces. Readers are referred to [18] for a recent review.

As mentioned before, the electric field in the crack cavity can significantly affect the solution; thus, it is essential to consider the electric field in the opened crack cavity. When the conventional boundary integral equation (CBIE) is applied to this kind of thin region, one concern is the nearly singular integrals that arise when the other boundary or interior points are close to, but not on, the boundary. Several calculation methods for nearly singular integrals have been proposed [19–27]. Motivated by the issues of electric fields in the crack cavity and nearly singular integrals, this paper describes an iterative approach based on sub-region BEM for analyzing cracks in two-dimensional piezoelectric media with exact boundary conditions.

2. Sub-region boundary element analysis of cracks in piezoelectric media

2.1. Sub-region model

We consider a central crack of length $2a$ in a piezoelectric rectangular medium of height $2h$ and width $2b$ (Fig. 1). We assume that a mechanical load $p(x_1)$ and an electric displacement load $q(x_1)$ along the x_2-axis are applied on the top and bottom surfaces of the rectangular plate (Fig. 1a). The two surfaces of the crack are traction-free. Under the applied loadings, a crack will open. The crack-cavity domain is denoted by Ω_c and its boundary is given by (Fig. 1b)

$$\Gamma_c = \Gamma_{c1} + \Gamma_{c2}, \tag{1}$$

where Γ_{c1} and Γ_{c2} are the upper and lower surfaces of the deformed crack, respectively. Thus, the entire rectangle can be divided into three sub-regions, two of which, denoted by Ω_1 and Ω_2, are bounded, respectively, by

$$\Gamma_1 = \Gamma_{12} + \Gamma_{1c} + B_1 + B_2 + B_3, \tag{2}$$

$$\Gamma_2 = \Gamma_{21} + \Gamma_{2c} + S_1 + S_2 + S_3, \tag{3}$$

where Γ_{12} and Γ_{21} are the common boundaries between regions Ω_1 and Ω_2, respectively; Γ_{c1} and Γ_{1c} (Γ_{c2} and Γ_{2c}) are the common boundaries between regions Ω_1 and Ω_c (Ω_2 and Ω_c), respectively, and B_1,B_2, and B_3 (S_1,S_2, and S_3) are the outer boundaries of region $\Omega_1(\Omega_2)$, as shown in Fig. 1b.

For the piezoelectric domain without any body force and current in the two-dimensional (2D) Cartesian coordinate system $x_i(i=1,\ 2)$, the equilibrium equations, kinematic equations, and the constitutive relations, respectively, are given by

$$\sigma_{ij,j} = 0, D_{i,i} = 0, \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), E_i = -\varphi_{,i}, \tag{5}$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k, D_i = e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k, \tag{6}$$

where σ_{ij}, ε_{ij}, D_i, and E_i denote the stresses, strains, electric displacements, and electric field strengths, respectively; $u_i(u_1=u,\ u_2=v)$ and φ are the elastic displacements and electric potential, respectively; and C_{ijkl},e_{ijk}, and κ_{ij} are the elastic constant, piezoelectric constant, and the dielectric constant, respectively. A subscript comma followed by index i denotes partial differentiation with respect to the coordinate x_i, with repeated indices i, j indicating summation from 1 to 2. The associated boundary conditions are discussed below.

In the crack cavity, only the electric field exists, and the governing equations are given by

$$D^c_{i,i} = 0, \quad E^c_i = -\varphi_{,i}, \quad D^c_i = \kappa^c E^c_i, \tag{7}$$

where the subscript "c" denotes the corresponding value in the crack cavity, e.g., air.