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A parametric study of the MLPG method for thermo-mechanical solidification analysis

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ABSTRACT

Based on the meshless local Petrov–Galerkin (MLPG) method, a thermo-elasto-plastic analysis of solidification problem is presented. The effect of significant parameters of the MLPG method, including the size and shape of sub-domain and support domain, nodal arrangement, nodal density and Gaussian points on the solution accuracy of the problems is investigated to determine their optimal values. The local weak forms are derived by considering a Heaviside step function as the test function. To interpolate the solution variables, the moving least-squares (MLS) approximation is applied. Using the effective heat capacity method, thermal analysis of the solidification process is performed. The von-Mises yield criterion and isotropic hardening model are employed for the elasto-plastic behavior, and material parameters are assumed to be temperature-dependent. To demonstrate the capability of the present method in solving solidification problems, the obtained results have been compared with the analytical and accurate finite element method solutions.

1. Introduction

Meshless methods have attracted a lot of attention in the past two decades, due to their ability to eliminate the burdensome effort of mesh generation. The meshless local Petrov–Galerkin (MLPG) approach, which was proposed by Atluri and Zhu in 1998 [1], is one of the most successful meshless methods. The main advantage of the MLPG method is that it considers local weak formulation of the problem, and does not require a background mesh, either for the interpolation of the solution variables or for evaluation of the integrals. Therefore, this approach is a “truly meshless” method and creating the trial and test functions is implemented by only using local nodes. The reason for flexibility of the MLPG method is twofold; neither it is necessary that the trial and test functions be selected from completely identical functional spaces, nor the physical size of the test and trial domains need to be the same. Based on different types of test functions considered in the weak formulation, six different types of MLPG methods were introduced by Atluri and Shen, which are labeled as MLPG1 to MLPG6 [2]. In the present work, the MLPG5 method, wherein the test function is the Heaviside step function, is utilized. This method eliminates the necessity of domain integration, and shows high robustness and precision in solving many engineering problems [3]. In the recent years, the MLPG method has been employed by many researchers in solving various types of engineering problems, such as convection-diffusion [4], elasticity [5–7], thermoelasticity [8], elasto-plasticity [9,10], thermoplasticity [11], and fracture [12–14] problems. Applications of the MLPG method in various fields of engineering and scientific problems have been reviewed in [15].

Since the thermo-mechanical analysis of the solidification problem is usually associated with many complications and difficulties [16], analytical solutions are inevitably presented with many simplifying assumptions. In this context, Weiner and Boley [17] and Tien and Kaump [18] provided analytical solutions of stress distribution in a solidification process, which are very useful for verifying the results of numerical methods. Various numerical methods have been used for thermo-mechanical analysis of solidification problem. Williams et al. [19] applied the finite element method (FEM) and elasto-viscoplastic constitutive model to analyze the thermal stress distribution in a solidifying body. Heinlein et al. [20] developed the thermo-mechanical boundary element procedure to determine the stress field in the one dimensional solidification problem. Calculation of stress and temperature fields in a solidifying media was performed and implemented by Zabaras et al. [21,22], by using a front tracking FE model. Control-volume model of fluid flow and finite difference method were used to calculate three-dimensional thermo-elastic stresses in die casting by Cross [23]. To obtain temperature and stress distributions in the continuous casting of steel, Li and Thomas [24] employed an elastic-viscoplastic creep constitutive equation by utilizing a thermo-mechanical FE model. Samantha and Zabaras [25] performed a coupled thermo-mechanical FE methodology to analyze thermal transport and segregation of solidified aluminum

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alloys. A collocated version of a segregated finite-volume procedure was developed by Teskeredžić et al. [26,27], to solve the fully coupled heat transfer, flow, and stress formation in complete casting process.

Despite the inherent capability of meshless methods in handling the phase change problems, so far, little use of them has been made in analysis of thermo-mechanical solidification problems. Vertnik et al. [28] solved a transient direct-chill aluminum casting problem with simultaneous material and interphase moving boundaries by using the explicit local radial basis function collocation method (LRBFCM). Simulation of continuous casting of steel in the curved strand by using the LRBFCM was developed by Vertnik and Šarler [29]. Sajja and Felicelli [30] utilized the element-free Galerkin method to simulate channel formation during directional solidification processes of a multicomponent Ni–Al–Ta–W alloy. A physical model for calculating the macrosegregation with mesosegragates in binary metallic casts was numerically solved by Kosec and Šarler [31], using the LRBFCM. A novel LRBFCM was applied to the problem of stress field calculation during low frequency electromagnetic field of direct-chill casting of aluminum alloys by Mavrič and Šarler [32]. Then they extended the LRBFCM to solve transient coupled thermoelasticity problems [33]. Zhang et al. [34] developed a combined thermo-mechanical mesh-free procedure for calculating the thermal stresses of continuous casting by using the finite point and MLPG methods. They employed the finite point method for the analysis of the solidification process and the MLPG method was applied only for the thermo-elasto-plastic analysis. Vaghefi et al. [35] performed a thermo-elasto-plastic MLPG analysis to study the effect of mushy zone thickness on residual stress formation in alloy solidification. However, they did not carry out any parametric study of the MLPG method.

The optimal choice of meshless method parameters is very important, because it can improve the convergence and accuracy of the method significantly [2,36]. Several studies have been performed to find the optimal values of MLPG method parameters in solving various problems. Atluri and Shen [3] investigated the effect of some significant parameters on solution accuracy in different types of MLPG methods. These parameters included size of sub-domain and support domain, nodal arrangement, and Gaussian points. They solved the Laplace and Poisson equations for the purpose of error estimation and convergence studies. Subsequently, they presented optimal values of some parameters of various MLPG methods in solving a fourth order partial differential equation [37]. Nie et al. [38] proposed a practical mathematics model to find the optimal radius for support of radial weights used in the MLS methods for the 4th order spline weight function. Using the MLPG1 method, a convergence study of a diffusion equation was performed to optimize the number of nodes in each support domain and the size of sub-domain by Sterk and Trobec [39]. Moussaoui and Bouziane [40] studied the effect of the sizes of support domain and sub-domain in solving a thin elastic plate problem by the MLPG1 method. However, the parametric study of the MLPG method has not yet been made in the thermo-mechanical analysis of solidification problems. Of course, such a study is feasible when both thermal and mechanical analyses of the problem are performed by the MLPG method.

In this paper, a thermo-elasto-plastic MLPG analysis with parametric study is performed for solidification problems. The effect of some important parameters, including the size and shape of sub-domain and support domain, nodal density, nodal arrangement (regular or irregular nodal distribution), and the number of Gaussian integration points on the solution accuracy of the MLPG method are also investigated and their optimal values are determined. In order to formulate the thermo-mechanical phase change problem, the local weak forms are developed by using a Heaviside step function as the test function and the moving least-squares (MLS) approximation is applied to interpolate the solution variables. The effective heat capacity method is adopted in the thermal analysis of the phase change process. In order to describe the elastoplastic behavior, the von-Mises yield criterion and isotropic hardening model are employed, and material parameters are assumed to be temperature-dependent. In the formulation of the problem, the small strain increment theory and the generalized plane strain condition are considered. The plastic strain increment is calculated according to the Prandtl–Reuss rule. In the present uncoupled nonlinear thermo-elasto-plastic meshless formulation, the transient temperature field is obtained and used as a thermal load for the calculation of the thermal stress distribution in the solidifying medium. To illustrate the validity and capability of the present meshless method in solving solidification problems, the obtained results have been compared with the analytical and accurate FEM solutions.

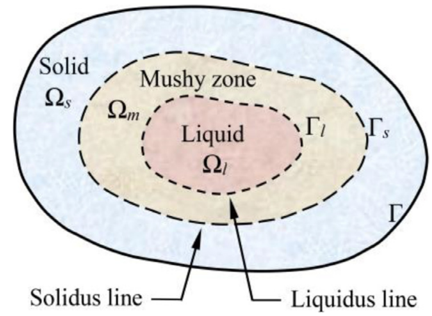

Fig. 1. The geometry and terminology of the alloy solidification problem.

2. Governing equations

A review of the governing equations and boundary/initial conditions of nonlinear transient heat conduction associated with phase change and also elasto-plastic problems are carried out in this section.

2.1. Governing equations of the thermal problem

A domain Ω with the boundary Γ, which is initially occupied by a liquid with temperature $T_0(\mathbf{x},0)$ is considered. The liquid is gradually cooled and the solidification process begins. In alloy materials, a mushy zone forms between the solid and liquid phases. The solid, liquid and mushy zone phases of the domain are denoted by Ω_s, Ω_l and Ω_m, respectively (see Fig. 1). Γ_s, represents the interface of the mushy zone and the solid phase while, Γ_l, denotes the interface of the mushy zone and the liquid phase.

Two classical approaches for thermal modeling of the solidification process have been the use of front tracking methods and fixed grid methods [41]. In front tracking methods, the energy balance is written separately for each phase and the grid should be updated to match the solid and liquid phases at each step. In these methods a special treatment for modeling discontinuities in piecewise homogeneous media is required in the case of higher order modeling like in meshless approximation. There are other approaches to overcome this problem, such as the use of double nodes on the interface of the two phases [42,43]. The difficulty of front tracking methods is that remeshing of the problem domain is needed during the solidification process. In fixed grid approaches, however, the energy balance equation is written for the whole domain [44]. The main advantage of these methods is the possible use of the weak formulation of the classical transient heat conduction problem.

Based on fixed grid methods, for the whole domain Ω, the single energy conservation equation is expressed in terms of the enthalpy function [45], as follows:

$$\left[k(\mathbf{x},T)T_{,i}(\mathbf{x},t)\right]_{,i} = \dot{H} \quad \text{in } \Omega, \tag{1}$$

where T, k, H, $\mathbf{x}$ and t are respectively, temperature, thermal conductivity, enthalpy function or the total heat content, position of a point in the domain, and time. The comma followed by the index i indicates the partial derivative with respect to the spatial coordinate x_i, and the superimposed dot denotes the partial derivative with respect to time t.