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Moving boundary analysis in heat conduction with multilayer composites by finite block method

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ABSTRACT

An inverse reconstruction investigation is presented to determine the inner boundary location (corrosion points) for the heat transfer in composite walls from measurement data on exterior boundary. Finite Block Method (FBM) is utilized in this paper to deal with transient heat problems across the multilayered composite walls. Starting from one-dimensional problems, Lagrange interpolation with equally spaced nodes is applied to create first order differential matrices and thereafter the higher order differential matrices are obtained. Then combining with mapping technique, physical domain is mapped into a normalized domain for two-dimensional or three-dimensional problems with 8 seeds or 20 seeds respectively. Both time-spatial approach and Laplace transform technique with Durbin's inversion method are employed in the simulating procedure. In addition, roots of Chebyshev polynomial of first kind are considered in FBM for the first time, which can improve the degree of convergence significantly. Three numerical examples are presented to validate the accuracy of FBM. Comparisons between Finite Element Method (FEM), FBM and Point Collocation Method (PCM) are demonstrated respectively. Numerical observation indicates that FBM has much higher degree of accuracy even with few collocation points.

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1. Introduction

Detecting the corrosion boundary of multilayer materials occurs extensively in engineering applications such as metallurgy. In the procedure of iron-steel-making, corrosion will take place on the inner surface of steel-smelting furnace which is always made of multilayer materials with different physical properties. We need to monitor the corrosion timely to prevent accidents. The corrosion degree inside steel-smelting furnace can be simulated through outermost layer information such as temperature or heat flux which may be measured easily. This kind of problem is always ill-posed and belongs to inverse problems.

The identification of corrosion boundary in heat transfer problems has been studied numerically by Aparicio and Atkinson [1], Bryan and Caudill [2,3] and Fredman [4]. Lots of numerical algorithms such as Finite Element Method (FEM), Finite Difference Method (FDM) as well as Boundary Element Method (BEM) are available for inverse problems in engineering, see Liu and Zhang [5], Raynaud and Bransier [6], Aliabadi and Wen [7]. Although FEM is considered as the most powerful and well developed tool in numerical engineering, advanced numerical methods have become more attractive recently such as meshless approaches. In the last decade, meshless methods based on interpolation techniques like Radial Basis Functions (RBF) and Moving Least Square (MLS) provide an efficient tool in engineering analysis, see Powell [8], Jackson [9], and Reinhard Farwig [10]. Method of Fundamental Solutions (MFS) developed as a useful technique in numerical computation by Mathon and Johnston [11] and then Golberg and Chen [12] extended its application further. Hon and Wei [13] utilized MFS to solve inverse heat transfer problems. Wei and Yamamoto [14] employed this method to identify moving boundary for heat conduction problems with Cauchy conditions. Li and Wei [15] proved the uniqueness of corresponding solutions. However, it is difficult to make further convergence study. Same problems with Stefan–Boltzmann conditions had been studied by Hu and Chen [16] and the existence of the corresponding solutions had been proved. Niu et al. [17,18] employed time-spatial method as well as Kansa method to solve inverse heat transfer problems in inhomogeneous media. In their study, time is treated as an extra coordinate and the method of discrete Tikhonov regularization is used due to ill-posed characteristics of inverse problems.

Differential Quadrature Method (DQM) developed by Bellman [19] is a powerful numerical method for lots of boundary problems. It is applied extensively in engineering and sciences by Bert et al. [20,21]. Liew et al. [22–26] proposed a lot of investigations on DQM in plate vibration as well as bending. When they use same number of discrete

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nodes, it can be found that DQM has better convergent results than FEM. Bert and Malik [27] gave a comprehensive review on DQM as well as its application. Recently Wen et al. [28], Li et al. [29,30] proposed a new meshless method called Finite Block Method (FBM) which is based on point collocation concept. It has been applied to solve heat transfer and elastodynamic problems in both 2D and 3D in Functionally Graded Materials (FGMs) with "star" performance. The novelty of FBM is that both first order and high order partial differential matrices in physical domain are based on the first order differential matrices constructed by Lagrange polynomial in normalized domain. Meanwhile, partial differential matrices of higher order are not essential in the governing equations with non-constant coefficients such as thermo-elasticity problems in inhomogeneous media. At the interface between two adjacent blocks in similar material, all stress components are found to be continuous [30].

The aim of this paper is to propose an inverse reconstruction procedure to determine the inner boundary (corrosion points) location in the problem of heat conduction for composite walls from measurement data of both temperature and heat flux on exterior boundary. FBM is extended to reconstruct the shape of corrosion boundary line (or surface) for 2D (or 3D) problems. First, with Lagrange interpolation where collocation points are distributed uniformly along a straight line, the first order differential matrix can be constructed for 1D problem, after which differential matrices of higher order can be obtained directly. Partial differential matrices for 2D and 3D problems can be evaluated similarly. Object with irregular boundary configuration in physical domain should be divided into quadratic blocks. With several quadratic shape functions, each block is mapped into a normalized square. In this procedure, elements with 8 (or 20) seeds are employed for 2D (or 3D) problems. Thereafter, the continuity conditions of heat flux and temperature along interface between two blocks should be satisfied and the partial differential operators in each block should satisfy a strong form too. Two approaches are proposed to deal with time dependent behavior in this paper, i.e. time-space approach (TS) where we do not distinguish time and space variables, and Laplace transform approach (LT), where the dependence on time variable t is reduced to the dependence on the transform variable s . For the second approach, in physical domain, the Durbin's inversion technique [31] for Laplace transformation is applied to get the nodal values.

The paper is organized as follows. Fundamentals of FBM which can be used directly to solve time independent problems are given in Sections 2 and 3. Time-space approach and Laplace transform approach are shown in Section 4 to deal with time dependent behavior. Three numerical examples are analyzed to demonstrate the procedure of FBM in identifying the moving boundary. Comparisons among FBM (TS), FBM (LT), FEM and RBF have been shown in Section 5. Finally a conclusion is presented in Section 6.

2. Lagrange interpolations for FBM

2.1. One dimension problems

For 1D problem in normalized domain, collocation points are equally spaced along ξ axis, as follows

$$\xi_i = -1 + \frac{2(i-1)}{M-1}, i = 1, 2....M, \tag{1}$$

where M denotes the number of discrete points on ξ-axis. By Lagrange polynomial interpolation, smooth function $u(\xi)(-1 \le \xi \le 1)$ can be approximated by

$$u(\xi) = \sum_{j=1}^{M} \prod_{k=1,k\neq j}^{M} \frac{\xi - \xi_k}{\xi_j - \xi_k} u_j, \tag{2}$$

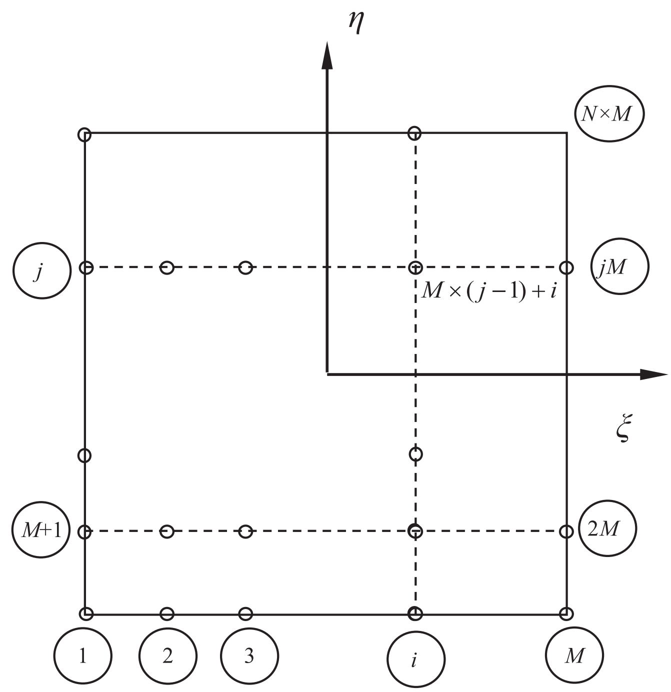

Fig. 1. Uniformly distributed nodes in normalized domain.

where $u_j = u(\xi_j)$ is the nodal value. Through (2), the first order derivative of $u(\xi)$ can be obtained

$$\frac{du}{d\xi} = \sum_{j=1}^{M} u_j \prod_{\substack{k=1,\\ k\neq j}}^{M} (\xi_j - \xi_k)^{-1} \sum_{i\neq j}^{M} \prod_{h=1,h\neq j,h\neq i}^{M} (\xi - \xi_h). \tag{3}$$

For all nodes, the first order derivative can be expressed, in matrix form, as

$$\mathbf{U}_\xi = \mathbf{D}_0 \mathbf{u}, \tag{4}$$

where $\mathbf{u} = [u_1, u_2, ...u_M]^T$ is vector of nodal values, $\mathbf{U}_\xi = [u'_1, u'_2, ..., u'_M]^T$ is vector of the first order derivative of $u(\xi)$ at each node. $\mathbf{D}_0$ is the first order differential matrix. For m-th order derivative, we have

$$\mathbf{U}_\xi^{(m)} = \mathbf{D}_0^m \mathbf{u}, m > 0, \tag{5}$$

where $\mathbf{U}_\xi^{(m)} = [u_1^{(m)}, u_2^{(m)}, ..., u_M^{(m)}]^T$ is vector of m-th order derivative of nodal value.

2.2. Multi-dimensions

Similar to 1D problems in normalized domain, we have $\eta_j = -1 + \frac{2(j-1)}{N-1}, j = 1, 2....N$,where N indicates the number of discrete points along η-axis. With Lagrange interpolation, smooth function $u(\xi, \eta)$ can be approximated as

$$u(\xi,\eta) = \sum_{i=1}^{M}\sum_{j=1}^{N} F(\xi,\xi_i) G(\eta,\eta_j) u_k = \sum_{k=1}^{Q} \psi_k(\xi,\eta) u_k, \tag{6}$$

where

$$F(\xi,\xi_i) = \prod_{\substack{m=1\\ m\neq i}}^{M} \frac{(\xi-\xi_m)}{(\xi_i-\xi_m)}, G(\eta,\eta_j) = \prod_{\substack{n=1\\ n\neq j}}^{N} \frac{(\eta-\eta_n)}{(\eta_j-\eta_n)}, \tag{7}$$

For 2D problems, collocation nodes in normalized domain are shown in Fig. 1. $u_k = u(\xi_k, \eta_k)$ indicates the nodal value at points P_k which stands for $P(\xi_k, \eta_k)$, where $k = (j-1) \times M + i$, $i = 1, 2, ..., M$ and

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