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A Stokes–Brinkman model of the fluid flow in a periodic cell with a porous body using the boundary element method

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ABSTRACT

The problem of viscous incompressible flow in a periodic cell with a porous body is solved. The Stokes flow model is adopted to describe the flow outside the body and the Brinkman equation is applied to find the filtration velocity field inside the porous domain. The conditions on the boundary between the free fluid and the porous medium for the porous body of arbitrary shape are obtained. The boundary value problem for the joint solution of the biharmonic and Brinkman equations for the stream functions outside and inside the porous body are then solved using a boundary element method. Good agreement of the numerical and analytical models for the Kuwabara circular cell model is shown for the fluid flow through a porous circular cylinder. The fluid flow past a circular, square, triangular cylinders and a circular body of uneven surface (an idealized model of a viral capsid) in a rectangular periodic cell are calculated. Comparison of the results obtained with the numerical solution from a CFD ANSYS/FLUENT model shows good accuracy of the developed mathematical model.

1. Introduction

The solution of the problem of fluid flow through porous bodies is used to describe many hydrodynamical processes in environmental and medical science. For example, such flows are found in aerosol filters and respirators and for particle shaped viruses in biofluids. In the case of aerosol filters porous bodies can be used as elements to increase the efficiency of the deposition of aerosol particles [1,2]. In order to calculate the two-phase flows of dusty air for such filters it is important to develop efficient mathematical models of fluid flow past porous bodies in periodic cells [3].

The fluid flow through a circular porous cylinder assuming potential flow of an incompressible fluid was modelled in work [4].

One of the problems with modelling fluid flow in domains containing porous medium is connected with the formulation of the boundary conditions on the interface between the free space and porous medium. The choice of the conditions depends on the mathematical model adopted. Various boundary conditions are studied in the works of Beavers and Joseph [5], Saffman [6], Neale and Nader [7], Haber and Mauri [8], Vafai and Thiyagaraja [9], Sahraoui and Kaviany [10], Ochoa-Tapia and Whitaker [11,12].

The analytical solution of the problem of the fluid flow past an isolated porous cylinder and a system of porous cylinders using a cell model was firstly obtained by Stechkina [13]. The cell model used was based on the widely adopted Kuwabara cell model [14] and included the Stokes flow model [15] outside and the Brinkman equation [16] inside the porous cylinder. The cell model with Kuwabara boundary conditions was also used by Deo et al. [17] and Kirsh [1] to determine the velocity field of the flow over and through a porous cylinder in the case of small Reynolds number flow using the analytical solution and the collocation method.

A review of analytical investigations of fluid flow past porous cylinders and spheres is given in the works of Deo et al. [18] and Vasin and Filippov [19]. Generally, the cell model is an approximate model of fluid flow and its accuracy depends on the porosity of the porous medium. To obtain an accurate fluid flow velocity field numerical models using the real array geometry should be adopted.

Viscous flow models for flows through porous bodies usually adopt the combination of the Stokes model in the free space and the Darcy or Brinkman model in the porous medium. Such models were used to study the viscous flow through isolated porous cylinders and spheres by Masliyah and Polikar [20], Nandakumar and Masliyah [21], Noymer et al [22], Vanni [23], Vainshtein et al [24,25].

The Navier–Stokes equations with Darcy and Forchmeier terms were solved numerically to simulate the fluid flow past a porous cylinder in Beckermann and Viskanta [26], Vafai and Kim [27], Basu and Khalili [28], Bhattacharyya et al [29].

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Nomenclature

D_{12}	components of strain rate tensor;
E_{vy}, E_ω	absolute errors;
f	arbitrary function;
h	radius of circular cell;
G_3, G_4	Green functions;
h_1, h_2	height and half width of rectangular cell;
H_1, H_2	Lame coefficients;
I_0, I_1	modified Bessel function of the first kind of zero and first order;
K_0, K_1	modified Bessel function of the second kind of zero and first order;
n, m	number of linear elements on boundaries Γ^e and Γ^i;
m_b	number of bumps of capsid model;
p	pressure;
q	curvilinear coordinate;
Q	coefficient of fluid capture;
(r, θ)	polar coordinates;
R_c	typical size of porous body;
s	boundary arc;
S	dimensionless parameter;
U	velocity scale;
v	velocity;
(x, y)	Cartesian coordinates;
(x_{ck}, y_{ck})	coordinates of segment centers.

Greek symbols

α	solidity;
β	interior angle at point on boundary;
γ	amplitude ratio of a bump;
Γ	boundary;
ε	porosity;
ε_{vy}, ε_ω	relative errors;
η	negative vorticity;
θ	the angle of the tangent to the current integration linear segment;
κ	permeability;
μ	fluid viscosity;
ρ	distance between current point and boundary point;
τ	shear stress;
ψ	stream function;
ω	vorticity;
Ω	domain.

Subscripts

a	analytical
r	radial
θ	tangential
x	cartesian x – component
y	cartesian y – component
1, 2	coordinate components

Superscripts

e	exterior;
i	interior;
$'$	derivative with respect to normal.

Using the Boundary Element Method (BEM) for solving the problem of fluid flow through porous bodies has advantages compared with the finite volume (FVM) or finite differences (FDM) methods due to its reduction of the dimension of the boundary value problem. Additionally adopting the BEM enables boundary value problems to be solved for porous bodies of any shape. Whilst the FVM and FDM have difficulties with the numerical solution of the fluid flow through porous bodies of complex shape. The majority of previous work in the area is devoted to the study of fluid flow through circular or rectangular bodies [3,30]. But there are applications where it is necessary to study the fluid flow through porous bodies of arbitrary shape (for example, the flow through aerosol fibrous filters containing deposit or the fluid through a viral capsid). Presently these problems remains unsolved. In this paper a mathematical model of fluid flow through arbitrary shaped porous bodies is developed using the BEM approach.

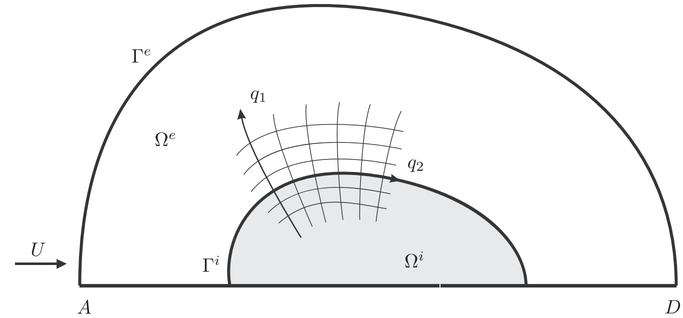

Fig. 1. Fluid flow domain.

The fluid flow past a periodic row past of arbitrary shaped porous bodies, is considered under the assumption of viscous incompressible flow. The approximate periodic circular [14] and rectangular cell models are used to formulate the fluid flow problem. The Stokes flow model is adopted outside the body in the cell and the Brinkman model inside the porous body to describe the flow velocity. The resulting boundary value problem for the stream function in the two domains with boundary conditions on the interface between the free space and the porous medium is formulated. The boundary conditions include the stream function and vorticity as variables and can be used for arbitrary curvilinear boundaries. The boundary element method (BEM) is then used to solve the boundary value problems. The numerical solution obtained is compared with the analytical solution in the circular cell case and good agreement of the two solutions is shown. The fluid flow past circular, square, triangular cylinders and a circular body with an uneven surface (an idealized model of a viral capsid) in a rectangular periodic cell is also calculated. Comparison of the numerical results with corresponding data obtained using CFD ANSYS/FLUENT (www.ansys.com) shows good correlation of the developed BEM model and FVM solution.

2. The problem statement

The two-dimensional flow of an incompressible fluid with speed U in a periodic cell with a porous body at a small Reynolds numbers is considered. The permeability κ of the porous medium is assumed to be constant. Due to the fluid flow symmetry we select as a calculation domain the upper part of the periodic cell that consists of $\Omega = \Omega^e \bigcup \Omega^i$ where Ω^e is the free fluid space and Ω^i the porous body domain (Fig. 1). The line AD is the symmetry axis. All quantities in the domains Ω^e and Ω^i are denoted by indexes e and i.

The fluid flow in the domain Ω^e limited by the boundary Γ^e is described by the Stokes model given by Eq. (1)

$$\nabla p^e = -\mu^e \operatorname{rot} \boldsymbol{\omega}^e, \quad (1)$$

where p^e is the pressure of outer flow, μ^e is the fluid viscosity, $\boldsymbol{\omega}^e = (0, 0, \omega^e)$ is the flow vorticity vector. The fluid flow in the porous domain Ω^i limited by the boundary Γ^i is described by the Brinkman model

$$\nabla p^i = -\frac{\mu^e}{\kappa}\boldsymbol{v}^i - \mu^i \operatorname{rot} \boldsymbol{\omega}^i, \quad (2)$$

where p^i is the pressure, $\boldsymbol{\omega}^i = (0, 0, \omega^i)$ is the vorticity, $\boldsymbol{v}^i$ is the average filtration velocity and μ^i is the fluid viscosity in the porous domain. The fluid viscosity in the free space and the porous body μ^e and μ^i differs due to the additional fluid drag in the porous medium.