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Mixed Discrete Least Squares Meshfree method for solving the incompressible Navier–Stokes equations

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ABSTRACT

A Mixed Discrete Least Squares Meshfree (MDLSM) method is proposed in this paper for the solution of incompressible Navier–Stokes equations. A semi-incremental two-step fractional projection method is first used to discretize the incompressible Navier–Stokes equations, followed by a mixed formulation used to solve the pressure equations. Using the mixed formulation, it is expected that the accuracy of the pressure approximation and in particular the pressure gradients are improved compared with that of conventional solution methods and in particular Discrete Least Squares Meshfree (DLSM) method. DLSM method is based on minimizing the least squares functional defined as the weighted summation of the squared residuals of the differential equation and its boundary conditions. The method is not subject to the Ladyzenskaja–Babuska–Brezzi (LBB) condition since it formulates the problem in the form of a minimization problem rather than a saddle-point problem. A number of numerical experiments are used to evaluate the efficiency of the proposed MDLSM method and to compare its accuracy against the DLSM method. From the results, it is found that the proposed MDLSM method can efficiently simulate the incompressible fluid flow problems. Furthermore, it can be concluded that the MDLSM method has higher accuracy compared with the DLSM method.

1. Introduction

Meshfree methods have attracted many researchers' attentions for solving the partial differential equations (PDEs) over the last few decades because of their merits compared to mesh-based methods. Although the mesh-based methods such as finite volume method (FVM) and finite element method (FEM) are widely used to simulate the engineering problems, these methods suffer from the inherent limitation of having to rely on meshes with a properly defined connectivity. Unlike the mesh-based methods, only a set of nodes is used in meshfree methods to discretize the problem domain. The meshfree methods, therefore, can be an efficient alternative to overcome the meshing difficulties in mesh-based methods [1].

The meshfree methods are normally categorized into two major groups regarding the approximation approach; namely kernel function method, and polynomial series method. Smoothed particle hydrodynamic (SPH) and moving particle semi-implicit (MPS) are the most known meshfree methods using the kernel approximation. The SPH method has been efficiently used to solve the flow problems such as free surface flows [2], viscous and heat conducting flows [3], two-fluid modeling [4], multiphase flow [5], and sediment scouring and flushing [6]. The MPS method is mainly similar to the SPH method. In this method the spatial derivatives are calculated without recording to the gradient of Kernel function. The method successfully has been applied for simulating the free surface [7], wave breaking [8] and multi-phase flow [9, 10] problems. The computational effort of the kernel function method is less than the polynomial method; however the consistency and accuracy of results are higher when the polynomial series method is used [1]. Furthermore, when polynomial series are used, higher-order accuracy and consistency can be achieved by increasing the order of basic functions. This property is more useful when dealing with the complex problems. The methods such as element free Galerkin (EFG), meshless local Petrov–Galerkin (MLPG), and DLSM are the methods using the series polynomial method.

Meshfree methods, which are based on the polynomial series approximation, fall into two major categories regarding the discretization form; namely weak-form and strong-form. In the weak-form formulations, required consistency of the trial function can be reduced via integration by parts. Such a property, which is also useful for improving the accuracy of the results, is absent in the strong-form formulations. Therefore, the required order of the basic function is lower in the weak-form compared to the strong-form. However, weak-form methods require back-

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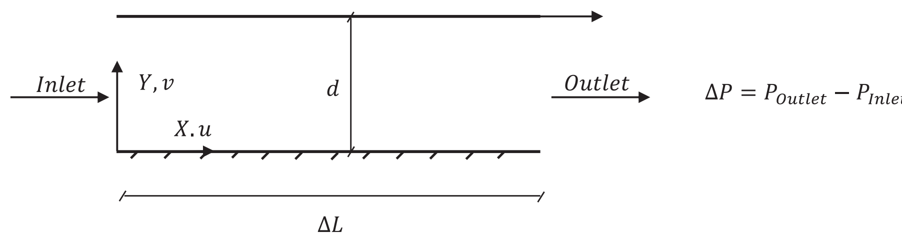

Fig. 1. Details of the combined Poiseuille and Couette flow problem.

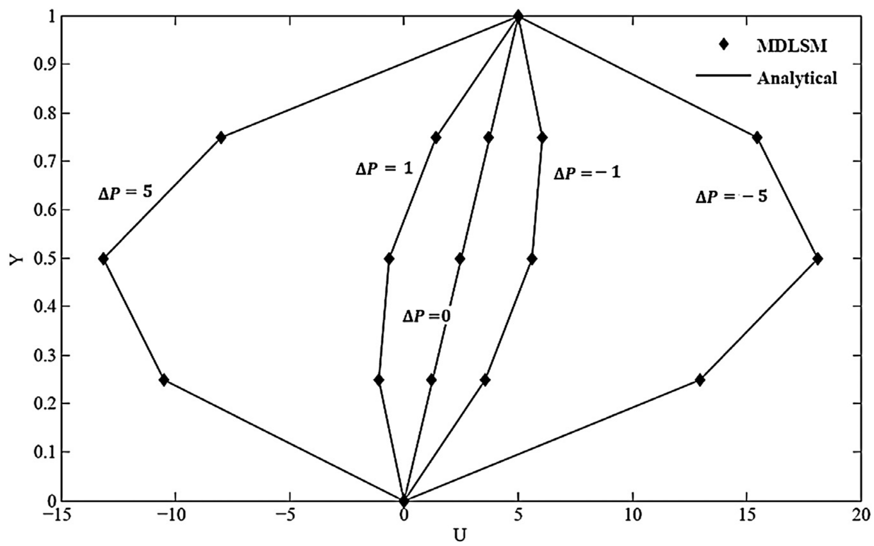

Fig. 2. Comparison of the MDLSM results with analytical solutions for combined Poiseuille and Couette flow.

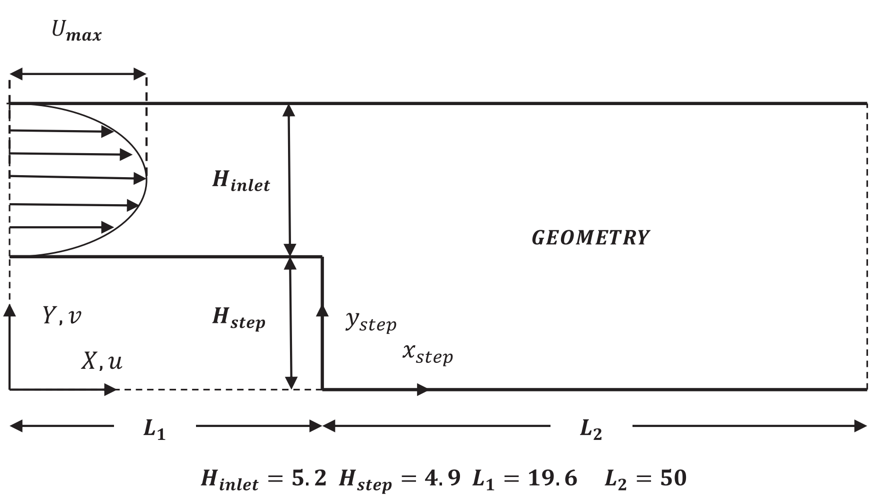

Fig. 3. Geometry of the backward facing step.

ground mesh for the numerical integration which undermines the meshfree properties of these methods. Furthermore, the numerical integration makes the weak-form methods computationally expensive. In fact, while the difficulties of meshing procedure in mesh-based methods are moderated by using the meshfree weak-form methods, such methods are not entirely independent of mesh [11]. On the other hand, the strong-form meshfree methods do not require the numerical integration so that such methods could be considered as truly meshfree.

The EFG [12] and MLPG [13] methods are mostly used meshfree methods which are formulated based on the weak-form. The EFG method has been used to investigate various flow problems such as three-dimensional Turing patterns [14] and unsteady heat transfer [15]. The MLPG method has also been efficiently used to simulate the solid mechanics problems [16, 17], convection–diffusion problems [18], and the incompressible Navier–Stokes equations [19]. The method is also stabilized for modeling the steady state incompressible fluid flows [20]. Although the MLPG method circumvents the use of background mesh by applying the local weak-form, the method suffers from the asymmetry of the coefficient matrix and numerical difficulties associated with the local numerical integration procedure near and/or on the boundaries.

Recently, DLSM method, as a strong-form meshfree method, was developed to solve different engineering problems such as elliptic partial differential equations [21], linear elasticity [22, 23], steady-state solution of incompressible Navier–Stokes equations [24], free surface prob-