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Algebraic formulation of nonlinear surface impedance boundary condition coupled with BEM for unstructured meshes

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ABSTRACT

This paper deals with nonlinear eddy currents problems introducing a novel surface impedance boundary conditions (SIBC). The SIBC is formulated in the algebraic framework giving to the field problem a circuital interpretation. In this way, the material behavior of the conductive computational domain is described by a network of lumped components. This paper shows that this matrix can be defined analytically in the case of linear magnetic characteristic or numerically when the material is nonlinear. It is shown that by mapping the magnetic nonlinearity into the circuit matrix the nonlinearity of the problem is smoothed and, therefore, a simple iterative scheme can be used. The SIBC is coupled to a BEM (for the region surrounding) giving rise to a hybrid solution. The method has been tested in terms of efficiency and accuracy. The use of the Schur complement of the solution matrix has allowed to decrease the solution time of an order of magnitude. A block triangular preconditioner is finally proposed in the case of highly distorted elements of the mesh. The preconditioner is based on the sparsification of the BEM matrix and its performance are analyzed considering a benchmark problem and a real problem (a wireless power transfer system).

1. Introduction

The computation of eddy currents in conductive media plays a key role in many industrial problems like induction heating [1,2], electric breaks [3,4], magnetic levitation [5], non destructive testing [6], etc. The standard technique for the study of these problems is the solution of the magneto-quasi static equations using the finite element method [7] or other volume-based methods [8]. These methods discretize the entire domain, made of sources, conductive workpieces and the surrounding open space. However, in many applications the conductive domain is limited in space, and more suitable techniques allow the use of open boundary with integral techniques. The FEM coupled with the boundary element method (BEM) is one of the most used hybrid schemes [9,10].

When the eddy currents are deliberately generated in workpieces, like in the case of induction heating, high frequency sources up to a hundred hertz are used to improve the coupling. In these cases, the penetration depth is limited with respect to the geometric dimensions of workpieces. This consideration makes the use of standard volume-based techniques not suitable, because they require a large amount of elements in the mesh to detect the small penetration depth. The surface impedance boundary conditions (SIBC) [11] have been introduced to effectively deal with these situations [12,13]. The main advantage of SIBC is the possibility of using analytical or semi-analytical formulations of the electromagnetic phenomena inside the conductive material, avoiding the fine discretization required by volume-based methods. For this reason, SIBC is effective and efficient in multi-scale, multi-domain problems, when the geometric or electric dimensions of objects in the domain can differ significantly [14,15].

SIBC is usually coupled with FEM, where source and open boundary regions are discretized [16]. More recently SIBC were coupled with the algebraic discretization techniques (like the finite integration technique and the cell method) [17,18].

In this paper an alternative coupling scheme is presented, where the SIBC scheme is coupled with the BEM. This approach is not new, since it has been introduced in the mid-'90s [19] limited to linear materials. A similar scheme has been used in [20], where the authors have proved that low and high order SIBC can be transformed in invariant forms depending only on the geometric parameters. However, the applicability of the technique is limited to linear problems. In [15], the authors have introduced the possibility of using nonlinear materials. The nonlinear domain is discretized in subregions, then the nonlinear volume equa-

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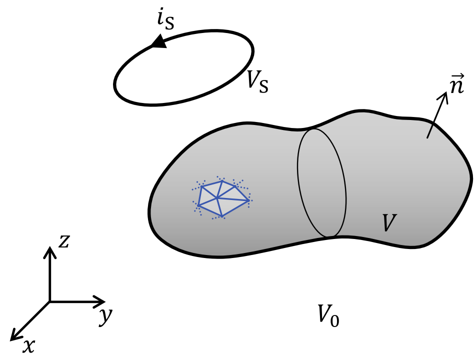

Fig. 1. Domain division: known sources V_S, eddy currents V and non-conductive V_0 regions.

tion and the surface integral equations are solved individually with an interactive procedure.

In this paper, the SIBC theory is reformulated in the context of the algebraic methods using two intertwined meshes, linked by duality relations. The interface conditions become circuit equations where electric voltages are defined on primal edges and currents on dual faces extending through the depth of the thin layer where current density is present [21]. The nonlinearity of the material is accounted integrating the contribution in the definition of nonlinear admittances. The preliminary results obtained in [21] for quadrilateral orthogonal meshes are extended to more general nonlinear problems on unstructured meshes.

2. Numerical formulation

2.1. Definition of the integral variables

The domain under study, represented in Fig. 1 is constituted by three complementary regions:

- the source domain, V_S with imposed currents, e.g., filamentary coils with known currents and/or massive conductors with known current density distribution;
- the domain V where eddy currents are induced, with finite conductivity σ and permeability μ_R (possibly nonlinear);
- the non-conductive ($\sigma = 0$ S/m) unbounded domain V_0, rigorously accounted by the Green's formula in the integral formulation.

The study is restricted to cases where the penetration depth (either in the linear and in the nonlinear cases) is at least 5 times smaller when compared to the geometric dimensions of the conductive region so that the current density profile is completely developed. Under this hypothesis, it is possible to consider only the boundary $S = \partial V$ of V. This boundary is discretized with a simplicial mesh $\mathcal{G}$, referred to as *primal mesh*, made of oriented faces, edges and nodes. From this mesh it is possible to derive a secondary mesh $\tilde{\mathcal{G}}$, namely *dual mesh*, obtained by barycentric subdivision of $\mathcal{G}$, where dual nodes are located in correspondence of the primal face barycenters, dual edges connect adjacent nodes and pass through the primal edge mid-points [22]. Dual faces are built onto these dual edges and extend through the depth of V until all the field quantities vanish. The orientation of the dual geometric entities in $\tilde{\mathcal{G}}$ is induced by the corresponding elements in $\mathcal{G}$. Fig. 2 shows these primal and dual complexes.

The electromagnetic variables are associated to these spatial entities as reported in Fig. 3:

- the electric voltage is defined along primal edges L:

$$e_k = \int_{L_k} \vec{E}_0 \cdot \mathrm{d}\vec{L}, \tag{1}$$

where $\vec{E}_0$ is the value of the electric field on the surface of the conductive object;

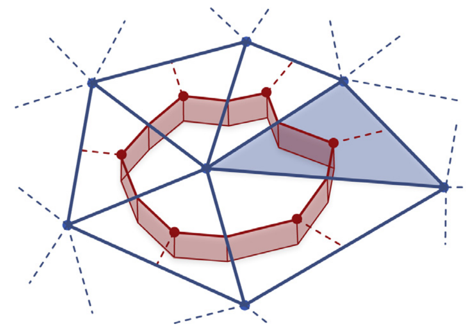

Fig. 2. Primal discretization (triangles) and dual discretization (barycentric division).

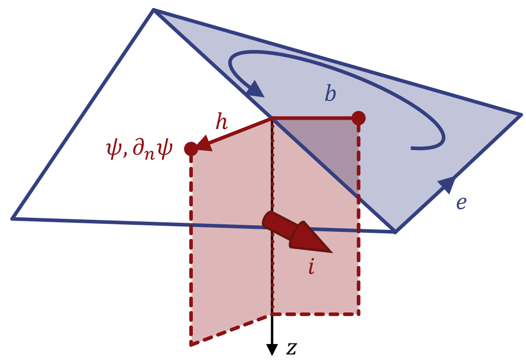

Fig. 3. Definition of the variables used for the SIBC–BEM formulation and their assignment to spatial elements. The primal mesh (in blue) lies on the boundary of the object, while the dual mesh extends through the volume depth. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

- the magnetic flux is defined through primal faces S:

$$b_m = \int_{S_m} \vec{B}_0 \cdot \mathrm{d}\vec{S}, \tag{2}$$

where $\vec{B}_0$ is the surface magnetic flux density;
- the magnetic scalar potential ψ is defined on dual nodes (barycenters of triangular faces), while its normal derivative with respect to the surface mesh $\frac{\partial \psi}{\partial n} = \partial_n \psi$ is assumed uniform over primal faces;
- the magneto-motive forces are defined along the dual edges $\tilde{L}$:

$$h_k = \int_{\tilde{L}_k} \vec{H}_0 \cdot \mathrm{d}\vec{L} \tag{3}$$

being $\vec{H}_0$ the surface magnetic field;
- the electric current i_k is obtained by integration of the current density $\vec{J}(z)$ through the generic dual face $\tilde{S}_k$:

$$i_k = \int_{\tilde{S}_k} \vec{J}(z) \cdot \mathrm{d}\vec{S} \tag{4}$$

It is worth noting that the current density $\vec{J}(z)$ depends on the position z along the depth of V to account for the magnetic diffusion.

2.2. BEM formulation

The fields in the homogeneous unbounded region V_0, external to the magnetic-conductive domain, are analyzed with a standard boundary element method (BEM) formulated in terms of reduced magnetic scalar potential ψ and its normal derivative $\partial_n \psi$ [23,24]. By defining the free space Green's function

$$G(\vec{r}, \vec{r}') = \frac{1}{4\pi|\vec{R}|}, \tag{5}$$

where $\vec{R} = \vec{r} - \vec{r}'$ is the vector from the source to the observation point, and applying the second Green's theorem to the Laplace equation $\nabla^2\psi =$