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The numerical manifold method for 2D transient heat conduction problems in functionally graded materials

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A B S T R A C T

Benefiting from the use of two cover systems, that is, the mathematical cover and the physical cover, the numerical manifold method (NMM) is capable of solving both continuous and discontinuous problems in the same platform. Presently, the NMM is further developed to tackle two-dimensional transient heat conduction problems in the functionally graded materials (FGMs). Firstly, the governing equation, the associated boundary conditions and the initial condition are presented. Then, the fundamentals of the NMM are briefly reviewed. Following, the NMM discrete formulations are derived based on the Galerkin-form weighted residual method and then solved with the backward difference scheme. Finally, for verification, three numerical examples with increasing complexity are tested on uniform mathematical covers composed of square mathematical elements, and our results well demonstrate the advantages of the proposed method in discretization and accuracy; besides, the effects of material gradient on the thermal behavior of FGMs are also examined.

1. Introduction

Due to the increasing demands in material performance, e.g., multifunction and better serviceability under complex working conditions, composite materials and structures are widely used in various areas such as aerospace, energy and automobile. Traditional composites like the laminated structures are not so satisfying due to the mismatch of physical properties across the material interfaces. The functionally graded materials (FGMs) are a new generation of composites where the volume fraction of the constituents changes gradually, producing a nonhomogeneous microstructure with continuously graded macro-properties such as the density, elastic modulus and thermal conductivity [1]. In view that the FGMs are generally designed to withstand elevated temperatures or large temperature gradients, the investigation of their thermal behaviors, especially those under transient state, is of great scientific and practical importance.

Seeing that the thermal parameters such as the heat conductivity and the specific heat may vary spatially, the analytical solutions to unsteady heat transfer problems in the FGMs are very limited. As alternatives, numerical approaches like the finite element method (FEM), the boundary element method (BEM), and the meshless methods, have been frequently adopted. Chen et al. [2] studied the unsteady heat conduction problems in the FGMs using the graded FEM and adaptive precise time integration scheme. Charoensuk and Vessakosol [3] applied the high order control volume FEM to investigate the transient heat transfer in the FGMs. Cao et al. [4] developed a hybrid graded FEM to compute the unstable temperature fields in the FGMs. Burlayenko et al. [5] obtained the transient temperatures and thermal stresses in the FGMs with the graded FEM. Sutradhar and Paulino [6] presented a simple BEM with boundary-only formulation for 3D unsteady heat conduction in the FGMs. Yang and Gao [7] computed the transient temperatures in the FGMs using a radial integration based BEM. Abreu et al. [8] proposed a convolution quadrature method based BEM to calculate the time-dependent temperatures in both homogenous materials and FGMs. Yu et al. [9] formulated a radial integral BEM together with the differential transformation technique to analyze transient heat conduction phenomenon in the FGMs. Sladek et al. [10] considered the unstable temperature fields in the FGMs with the meshless local boundary integral equation method. Using a higher-order plate theory and a meshless local Petrov–Galerkin method, Qian and Batra [11] investigated the 3D heat conduction problems in functionally graded thick plate. Through an improved meshless radial point interpolation method, Khosravifard et al. [12] examined the nonlinear unsteady heat transfer problems involving heat sources. Krahules et al. [13] adopted the meshless local radial basis function method to conduct stationary and transient heat conduction analysis in 2D and 3D FGMs.

Recent years, the numerical manifold method (NMM) [14] has been attracting more and more attention attributing to its excellent ability in solving continuous problems, discontinuous problems and also the tran-

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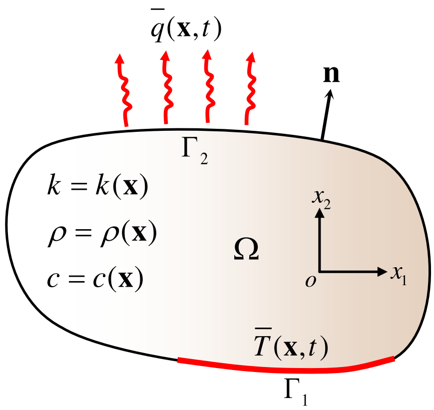

Fig. 1. Transient heat conduction in an isotropic FGM body.

sition from continuities to discontinuities. The superiority of the NMM originates from its bi-cover systems, i.e., the mathematical cover (MC) and the physical cover (PC). Accordingly, the major features of the NMM can be summarized as: (1) the MC may be inconsistent with all the physical boundaries, which may greatly facilitate the discretization procedure, especially for problems with complex geometrical configurations; (2) local physical characteristics can be properly captured in essence or manifested through the incorporation of certain special terms into the associated local functions; (3) higher-order approximations can be obtained by using higher-order local functions with the MC unchanged. In the past two decades, extensive efforts have been made to the application and development of the NMM in various fields, e.g., in fracture analysis [15–29], fluid flow [30–34], wave propagation [35–37], stability of rock mass [38–42] and phase change [43]. Besides, the NMM has also been extended to perform thermal analysis in homogeneous materials. Zhang and his coauthors tackled the steady [44] and transient [45] thermoelastic fracture behavior of 2D solids by the NMM. Gao and Wei [46] solved the 2D transient heat conduction problems with the complex variable meshless NMM. Zhang et al. [47] acquired the unsteady thermal fields by the NMM on Wachspress polygonal elements.

At present, in the light of the advantages of the NMM and the significance of FGMs thermal behavior studies, the NMM is further explored to analyze 2D transient heat transfer problems in the FGMs. To this end, the rest of the paper is organized as follows. In Section 2, the governing equation, the boundary conditions and initial condition are listed. In Section 3, following a brief introduction to the NMM, the discrete formulations of the concerned problems by the NMM are derived, and then some details about the solving procedures are described. To verify the proposed method, three typical numerical examples are tested in Section 4. Finally, the concluding remarks are drawn in Section 5.

2. Governing equation

Fig. 1 illustrates a transient heat conduction problem in a 2D body composed of the isotropic FGM. The physical domain Ω is enclosed by the contour $\Gamma = \Gamma_1 \cup \Gamma_2$, with Γ_1 and Γ_2, respectively, the temperature boundary and the heat flux boundary. The governing equation for this problem is

$$\frac{\partial}{\partial x_1}\left(k(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(k(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_2}\right) + Q = \rho(\mathbf{x})c(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial t} \tag{1}$$

where ∂ denotes partial derivative. k, ρ and c are, respectively, the thermal conductivity, the density and the specific heat at constant pressure of the FGM and may vary spatially with $\mathbf{x} = (x_1, x_2)$. T, t and Q denotes, respectively, the temperature, the time and the heat source.

The associated essential and natural boundary conditions are

$$T(\mathbf{x},t) = \bar{T}(\mathbf{x},t) \quad \text{on } \Gamma_1 \tag{2}$$

$$-k(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_1}n_1 - k(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_2}n_2 = \bar{q}(\mathbf{x},t) \quad \text{on } \Gamma_2 \tag{3}$$

where $\bar{T}$ and $\bar{q}$ are, respectively, the given temperature on Γ_1 and the heat flux on Γ_2. $(n_1, n_2) = \mathbf{n}$ is the outward unit normal to the domain in Fig. 1.

As for the initial condition, it is written as

$$T(\mathbf{x},t)|_{t=0} = T_0(\mathbf{x}) \tag{4}$$

3. Transient heat conduction analysis of the FGMs by the NMM

3.1. A brief introduction to the NMM

The NMM, proposed by Shi [14] as the extension and promotion of the discontinuous deformation analysis [48], was originally developed to efficiently and accurately capture the deformation behavior of rock mass, and has been widely adopted in other areas as mentioned in Section 1. In view that one of the major differences between the NMM and the well-known FEM lies in preprocessing, the general NMM discretization procedure is presented herein. To discretize a physical domain by the NMM, we firstly construct an MC, which is formed by a certain amount of mathematical patches (MPs) and should be large enough to cover the whole domain. As for the MPs, they may be composed of arbitrarily-shaped mathematical elements and may also be overlapped. Following, the corresponding physical patches (PPs) can be produced through the intersection of the MPs and the physical domain. The collection of the PPs then establishes the PC. Finally, after the intersection operation of as many as possible PPs, each non-overlapped section in all the PPs is just a manifold element (ME).

To make the above process clearer, the discretization of a rectangular physical domain in Fig. 2a is illustrated. To start with, an MC made up of a triangular MP M_1 and a circular MP M_2 in Fig. 2b is chosen to cover the rectangle. The intersection of the domain and the MPs generates the PC, which includes two PPs, i.e., P_1 and P_2 in Fig. 2c. The two PPs finally give three MEs, that is, E_1, E_2 and E_3 in Fig. 2d.

After the above procedure, the NMM approximation can be obtained. For the present problems, the transient temperature on the ME E is expressed as

$$T^h(\mathbf{x},t) = \sum_{i=1}^{n} w_i(\mathbf{x})T_i^P(\mathbf{x},t) \tag{5}$$

where n is the number of PPs shared by E and $w_i(\mathbf{x})$ is the partition of unity weight function on the ith PP [18]. $T_i^P(\mathbf{x},t)$ represents the local function defined on the ith PP, and for continuous PPs (e.g., PPs without crack tips or materials interfaces), it is frequently adopted as

$$T_i^P(\mathbf{x},t) = \mathbf{P}(\mathbf{x})\mathbf{a}_i(\mathbf{x},t) \tag{6}$$

where $\mathbf{a}_i$ is the column vector of the unknowns defined on the ith PP. $\mathbf{P}(\mathbf{x})$ is the row vector of polynomial basis taken as

$$\mathbf{P}(\mathbf{x}) = \left[1,\ x_1,\ x_2,\ x_1^2,\ x_1x_2,\ x_2^2,\ \ldots\right] \tag{7}$$

3.2. NMM formulations

Presently, the NMM discrete equations for the concerned problems are derived by the weighted residual method in Galerkin form [49]. By using the penalty method to enforce the essential boundary condition in Eq. (2) (it is noted that although the frequently used penalty method was adopted herein, other techniques such as the Lagrange multiplier method [50] and the scheme proposed in [51] are also applicable), the equivalent integral of Eqs. (1)–(3) is given by

$$\int_\Omega \varphi\left[\rho c\frac{\partial T}{\partial t} - \frac{\partial}{\partial x_1}\left(k\frac{\partial T}{\partial x_1}\right) - \frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right) - Q\right]d\Omega + \int_{\Gamma_1}\varphi_1\lambda\left(T - \bar{T}\right)d\Gamma + \int_{\Gamma_2}\varphi_2\left(k\frac{\partial T}{\partial x_1}n_1 + k\frac{\partial T}{\partial x_2}n_2 + \bar{q}\right)d\Gamma = 0 \tag{8}$$