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An inverse boundary element method computational framework for designing optimal TMS coils

Clemente Cobos Sánchez [a,*], Francisco Javier Garcia-Pacheco [b], Jose Maria Guerrero Rodriguez [a], Justin Robert Hill [c]

[a] Departamento Ingeniería de Sistemas y Electrónics, Avenida de la Universidad, 10, E-11519 Puerto Real Cádiz, Spain
[b] Departamento de Matemáticas, Avenida de la Universidad, 10, E-11519, Puerto Real Cádiz, Spain
[c] Department of Mathematics, Temple College, Temple Texas, 76504 USA

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A B S T R A C T

An inverse boundary element method and efficient optimisation techniques were combined to produce a versatile framework to design optimal TMS coils. The presented approach can be seen as an improvement and extension of the work introduced by Cobos Sanchez et al. [1] where the optimality of the resulting coil solutions was not guaranteed. This new numerical framework based on a constant boundary element method has been efficiently applied to produce optimal TMS coils with arbitrary geometry, allowing the inclusion of new coil features in the design process, such as optimised maximum current density or reduced temperature. Even the structural properties of the human head were considered using this approach at the design stage to produce more realistic TMS stimulators. Several examples of TMS coils were designed and simulated to demonstrate the validity of the proposed boundary element method approach, and the obtained results show that the described method is an efficient tool for the design of optimal TMS stimulators, which can be applied to a wide range of coil geometries and performance requirements considering the natural variability in the human head properties.

1. Introduction

Transcranial Magnetic Stimulation (TMS) is a non-invasive technique to stimulate the brain, which is applied to studies of cortical effective connectivity, presurgical mapping, psychiatric and medical conditions, such as major depressive disorder, schizophrenia, bipolar depression, post-traumatic, stress disorder and obsessive-compulsive disorder, amongst others [2].

In TMS, strong current pulses driven through a coil are used to induce an electric field stimulating neurons in the cortex. The efficiency of the stimulation is determined by coil geometry, orientation, stimulus intensity, depth of the targeted tissue and some other factors, such as stimulus waveform and duration.

The TMS stimulator most commonly employed is the so called figure-of-eight or butterfly coil, but since the invention of this technique numerous coil geometries have been proposed to improve the performance and spatial characteristics of the electromagnetic stimulation [3].

The problem in TMS coil design is to find optimal positions for the multiple windings of coils (or equivalently the current density) so as to produce fields with the desired spatial characteristics and properties [4,5] (high focality, field penetration depth, low inductance, low heat dissipation, etc.).

In engineering similar problems exist to the one found in TMS coil design, specifically, problems where one needs to determine a quasi-static spatial distribution of electric currents flowing on a conductive surface subjected to electromagnetic constraints. Some of these problems have been successfully solved by modelling the current under search in terms of the stream function using a boundary element method (BEM). A relevant application can be found in magnetic resonance imaging (MRI), where gradient coils have been efficiently designed following this technique [6,7].

The first effort to incorporate this numerical strategy to formulate a TMS coil design technique was presented by Cobos Sanchez et al. [1], in which a stream-function based current model is incorporated into an inverse boundary element method (IBEM). In that work, the desired current distribution is eventually obtained by solving an optimization problem, one in which a cost function or functional formed with a weighted linear combination of all the objectives, is minimized using classical techniques, such as simple partial derivation.

The computational approach in [1] demonstrated flexibility for the inclusion of new coil features in the design process, such as minimiza-

* Corresponding author.
E-mail address: clemente.cobos@uca.es (C. Cobos Sánchez).

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tion of the magnetic stored-energy, minimization of power dissipation or minimization of the undesired electric field induced in non target regions of the cortex. Unfortunately, despite the efficiency of the TMS stimulators designed using the stream function IBEM in [1], it was not known how optimal these coil solutions were. Especially since the associated optimisation involved a maximisation problem, which has to be rigorously tackled so as to produce the most effective stimulation of the desired cortex regions.

More recently, Koponen et al. [5] have also made use of a stream IBEM to develop another method for designing TMS coils of any desired overall shape and size, where the stored energy and focality can be controlled. Although the stimulators produced in [5] have a remarkable performance, they exhibit areas of high winding density over the region of stimulation. These dense portions of return windings are associated to high peak temperatures, and may also lead to unpractical designs, specially in TMS coils that are constructed from finite sized wire where there is a minimum wire separation that can be built.

The development of techniques capable of spreading the closest wires would be therefore beneficial to improve the buildability and thermal behaviour of coils designed by using a stream IBEM.

On the other hand, applications of TMS for diagnostic and therapeutic purposes are constantly growing, being often restricted by technical limitations [2]. The ability of the BEM to solve heat [8–11] and vibration [12] problems, along with the versatility of stream function based techniques opens up the possibilities of overcoming some of these restrictions with the design of a new generation of TMS stimulators with improved performance and novel properties, such as reduced mechanical stress, minimum coil heating, optimized maximum current density amongst others.

Nonetheless, most of these new performance features increase the mathematical complexity of the TMS coil design, and prompt the need to consider a robust computational framework to rigorously describe the problem and more efficient optimisation techniques, as classical approaches can no longer be straightforwardly applied to handle new non-linear requirements.

In this work, the numerical approach in [1] is improved to produce a computational optimisation framework for designing truly optimal TMS coils of arbitrary shape with novel performance properties such as, for instance, reduced coil heating or optimized current density; the latter is used to illustrate how the buildability of a stimulator designed in [5] can be increased.

Moreover, the suggested method here allows the structural properties of the human head to be considered in the design process to produce more realistic TMS stimulators.

The presented numerical approach is a combination of general optimisation techniques with a stream function IBEM, which permits the modelling of most of the TMS coil performance features as convex objectives.

The structure of this work is as follows. Firstly an outline of the stream function IBEM is presented in Section 2, which leads to the formulation of the TMS coil design problem in Section 3. Finally we illustrate the validity of this IBEM approach with the design and performance evaluation of several examples of TMS stimulators of different geometries, which have been chosen to demonstrate the suggested method, to elucidate the behaviour of new TMS coil requirements and how they can be use to improve performance and buildability.

2. Numerical model

2.1. The current density

A model of the current under search can be achieved by using a constant boundary element method (BEM), that allows the current distribution to be defined in terms of the nodal values of the stream function and elements of the local geometry (see [13]). So let us assume that the surface, $S \subseteq \mathbb{R}^3$, on which we want to find the optimal current, is divided into T triangular flat elements with N nodes, which are lying at each vertex of the element. If we consider the barycenters of the mesh triangles as $\mathbf{R_T} = \{\mathbf{r}_1, \dots, \mathbf{r}_T\}$, the current density at each element can be written as

$$\begin{array}{llll}\mathbf{J}: & \mathbf{R_T} \times \mathbb{R}^N & \rightarrow & \mathbb{R}^3 \\ & (\mathbf{r}, \psi) & \mapsto & \mathbf{J}(\mathbf{r}, \psi) \approx \sum_{n=1}^{N} \psi_n\, \boldsymbol{J}^n(\mathbf{r}),\end{array} \tag{2.1}$$

where $\psi = (\psi_1, \psi_2, \dots, \psi_N)^T$ is the vector containing the nodal values of the stream function and $\boldsymbol{J}^n : \mathbf{R_T} \rightarrow \mathbb{R}^3$ are functions related to the curl of the shape functions [13] known as current elements. In the following, $\psi \in \mathbb{R}^N$ is going to be the optimization variable.

If we denote by j_x^n, j_y^n, j_z^n to the Cartesian components of $\boldsymbol{J}^n$, then

$$\mathbf{J}(\mathbf{r}, \psi) \approx \sum_{n=1}^{N} \psi_n\, \boldsymbol{J}^n(\mathbf{r}) = \left(\sum_{n=1}^{N} \psi_n\, j_x^n(\mathbf{r}), \sum_{n=1}^{N} \psi_n\, j_y^n(\mathbf{r}), \sum_{n=1}^{N} \psi_n\, j_z^n(\mathbf{r}) \right)$$

and the absolute current density is $j(\psi) = (j(\mathbf{r}_1, \psi), \dots, j(\mathbf{r}_T, \psi))^T$ where

$$j(\mathbf{r}, \psi) := \sqrt{\left(\sum_{n=1}^{N} \psi_n\, j_x^n(\mathbf{r}) \right)^2 + \left(\sum_{n=1}^{N} \psi_n\, j_y^n(\mathbf{r}) \right)^2 + \left(\sum_{n=1}^{N} \psi_n\, j_z^n(\mathbf{r}) \right)^2}.$$

The use of this current model allows the discrete formulation of all the magnitudes involved in the problem. An appropriate boundary integral formulation of these magnitudes can be found in Appendix C (C.5), which allows to produce the following matrix equations that transform ψ to the various coil properties and objectives can be then constructed.

2.2. The magnetic field

The magnetic field at a series of H points, $\mathbf{r}_{\mathcal{H}} = \{\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_H\}$

$$b_{x_i}(\mathbf{r}_{\mathcal{H}}, \psi) = B_{x_i}(\mathbf{r}_{\mathcal{H}})\psi, \quad b_{x_i} \in \mathbb{R}^H, \quad B_{x_i} \in \mathbb{R}^{H \times N}, \; x_i = x, y, z. \tag{2.2}$$

The coefficient $B_{x_i}(h, n) = b_{x_i}^n(\mathbf{r}_h)$, is the x_i-component of the magnetic induction produced by the current element associated to the n^{th}-node in the prescribed h^{th}-point in $\mathbf{r}_{\mathcal{H}}$.

2.3. The stored energy in the coil

$$W(\psi) = \psi^T L \psi, \quad L \in \mathbb{R}^{N \times N}, \tag{2.3}$$

where L is the inductance matrix, which is a full symmetric matrix, and since the amount of stored magnetic energy is always a positive

$$\psi^T L \psi > 0, \quad \forall \psi \in \mathbb{R}^N, \quad \psi \neq 0 \tag{2.4}$$

then L is positive definite.

2.4. The resistive power dissipation of the coil

$$P(\psi) = \psi^T R \psi, \quad R \in \mathbb{R}^{N \times N}. \tag{2.5}$$

where R is the resistance matrix, which is also symmetric and positive-definite. Moreover, the power dissipation can be related to the current at the surface as $R \propto J_x^T J_x + J_y^T J_y + J_z^T J_z$.

2.5. The electric field

The electric field induced in a series of M points inside of the conducting system [7], $\mathbf{r}_{\mathcal{M}} = \{\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_M\}$

$$e_{x_i}(\mathbf{r}_{\mathcal{M}}, \psi) = E_{x_i}(\mathbf{r}_{\mathcal{M}})\psi, \quad e_{x_i} \in \mathbb{R}^M, \quad E_{x_i} \in \mathbb{R}^{M \times N}, \; x_i = x, y, z. \tag{2.6}$$

where the coefficient $E_{x_i}(h, n)$, is the x_i-component of the electric field induced by the current element associated to the n^{th}-node in the prescribed m^{th}-point in the conducting system $\mathbf{r}_{\mathcal{M}}$. It is worth noting that Eq. (2.6) can also be used to described the electric field induced in prescribed multi-compartment volume conductor made of different homogeneous sub-domains [14] with different electrical properties, as a representation of a heterogeneous system.