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RIBEM for 2D and 3D nonlinear heat conduction with temperature dependent conductivity

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ABSTRACT

In this paper, a new and effective radial integration boundary element method (RIBEM) is presented to solve nonlinear heat conduction with temperature dependent thermal conductivity of materials. Boundary-domain integral equation is formulated for nonlinear heat conduction by utilizing the fundamental solutions for the corresponding linear heat conduction, which contains a domain-integral due to the temperature dependence of the thermal conductivity of the materials. Two different approaches based on the radial basis functions are implemented to approximate the unknowns appearing in domain integrals. The arising domain-integral is converted into the equivalent boundary integrals using the radial integration method (RIM), resulting in a pure boundary element analysis algorithm. Newton–Raphson iterative method is applied to solve the final system of algebraic equations after the discretization. Numerical examples are presented to demonstrate the accuracy and the efficiency of the present method.

1. Introduction

Nonlinear heat transfer analysis is very important for many practical engineering areas [1–3]. Since it is very difficult to find analytical solutions, numerical methods are widely used for the analysis, such as finite element method (FEM), finite difference method (FDM) and finite volume method (FVM). The FEM is a well-established tool for analyzing non-linear and non-homogenous problems in engineering. Nevertheless, it is very time-consuming, which limits the application of the FEM method to some complex problems. To avoid the mentioned difficulties, boundary element methods (BEM) have been established and developed during the past two decades. The traditional boundary integral equations dealing with non-homogeneous [1,2] and non-linear heat conduction problems [3–9] include domain integrals in the ultimate integral equations. In order to evaluate these domain integrals, the computational domain region is required to be discretized into internal cells, making BEM lose its distinct advantage of boundary-only discretization. To circumvent this difficulty, some methods of transforming domain integrals into equivalent boundary integrals are frequently used. In these methods, the dual reciprocity method (DRM) developed by Brebbia [10] is extensively used. However, DRM requires particular solutions for the basic functions, which restricts its application to the complicated cases. Recently, a new transformation methodology, the radial integration method (RIM), has been proposed by Gao [11], which not only can convert any complicated domain integrals into the equivalent boundary in a unified way without using any particular solutions, but also can remove a variety of singularities appearing in the domain integrals. Due to the advantages of RIM, that particular solutions are not needed and various domain integrals appearing in the same integral equation can be handled simultaneously, RIM-based boundary element methods have won a good favor from many BEM researchers [12–15] in recent years. However, although the radial integration boundary element method (RIBEM) is very powerful to deal with the general non-linear mechanics problems [16] and non-homogeneous problems [17–24], there is no report about nonlinear heat conduction problems with temperature dependent conductivity using RIBEM.

In this paper, a novel type of 2D and 3D boundary-domain integral equation for nonlinear heat conduction problems is developed based on the fundamental solution for linear heat conduction problems. The arising domain-integral are transformed to the boundary using RIM by expressing the variable of temperature as a series of basic functions. To approximate the unknown functions, two different approaches based on the radial basis functions are implemented. Newton–Raphson iterative method is applied to solve the final system of algebraic equations. Some 2D and 3D examples are presented to demonstrate the accuracy of the present method.

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2. Boundary-domain integral equations for heat conduction problems with temperature dependent conductivity

The governing equation for heat conduction problems in isotropic nonhomogeneous media with temperature dependent thermal conductivity can be expressed as follows:

$$\frac{\partial}{\partial x_i}\left(k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}\right)+Q(\mathbf{x})=0 \tag{1}$$

where, x_i is the ith component of the spatial coordinates at point $\mathbf{x}$, $T(\mathbf{x})$ the temperature, $k(T(\mathbf{x}))$ the temperature dependent thermal conductivity at point $\mathbf{x}$ and $Q(\mathbf{x})$ is the heat-generation rate. The repeated subscript i indicates the summation through its range which is 2 for 2D and 3 for 3D problems.

Boundary conditions are given as follows:

$$\begin{aligned} T(\mathbf{x}) &= \bar{T}(\mathbf{x}) \\ q(\mathbf{x}) &= -k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial \mathbf{n}} \end{aligned} \tag{2}$$

In order to derive the boundary integral equation, the weight function $G(\mathbf{x}, \mathbf{y})$ is presented to Eq. (1) and the following domain integral item can be written:

$$\int_\Omega G(\mathbf{x},\mathbf{y})\frac{\partial}{\partial x_i}\left(k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}\right)d\Omega(\mathbf{x})+\int_\Omega G(\mathbf{x},\mathbf{y})Q(\mathbf{x})d\Omega(\mathbf{x})=0 \tag{3}$$

where, Ω represents the domain of the problem of interest.

Using Gauss's divergence theorem, the domain integral can be rewritten as follows:

$$\begin{aligned}\int_\Omega G\frac{\partial}{\partial x_i}\left(k(T)\frac{\partial T}{\partial x_i}\right)d\Omega &= \int_\Gamma Gk(T)\frac{\partial T}{\partial x_i}n_i d\Gamma-\int_\Gamma k(T)T\frac{\partial G}{\partial x_i}n_i d\Gamma \\ &+\int_\Omega T\frac{\partial G}{\partial x_i}\frac{\partial k(T)}{\partial x_i}d\Omega+\int_\Omega k(T)T\frac{\partial}{\partial x_i}\left(\frac{\partial G}{\partial x_i}\right)d\Omega\end{aligned} \tag{4}$$

Where, Γis the outer boundary of the domain Ω and n_i is the ith component for the outward normal vector n to the boundary Γ.

If the weight function $G(\mathbf{x}, \mathbf{y})$ is the Green's function for Laplace equation which satisfies the following equation:

$$\int_\Omega k(T(\mathbf{x}))T(\mathbf{x})\frac{\partial}{\partial x_i}\left(\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial x_i}\right)d\Omega(\mathbf{x})=-k(T(\mathbf{y}))T(\mathbf{y}) \tag{5}$$

then by substituting the relation into Eq. (4) and the result into Eq. (3), it follows that:

$$\begin{aligned}k(T(\mathbf{y}))T(\mathbf{y}) = &-\int_\Gamma G(\mathbf{x},\mathbf{y})q(\mathbf{x})d\Gamma(\mathbf{x})-\int_\Gamma k(T(\mathbf{x}))\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial \mathbf{n}}T(\mathbf{x})d\Gamma(\mathbf{x}) \\ &+\int_\Omega G(\mathbf{x},\mathbf{y})Q(\mathbf{x})d\Omega(\mathbf{x})+\int_\Omega \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial x_i}\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})d\Omega(\mathbf{x})\end{aligned} \tag{6}$$

where $q(\mathbf{x})$ is the heat flux

$$q(\mathbf{x})=-k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}n_i(\mathbf{x}) \tag{7}$$

The Green's function $G(\mathbf{x}, \mathbf{y})$ in Eq. (5) is

$$G(\mathbf{x},\mathbf{y})=\begin{cases}\frac{1}{2\pi}\ln\left(\frac{1}{r}\right) & f\ \text{or 2D problems} \\ \frac{1}{4\pi r} & f\ \text{or 3D problems}\end{cases} \tag{8}$$

where r is the distance between the source point $\mathbf{y}$ and the field point $\mathbf{x}$.

Eq. (6) is the boundary integral equation for the steady state heat conduction with temperature dependent thermal conductivity. It is effective only for internal points. For boundary points, a similar integral equation can be acquired by letting $y\to\Gamma$ as is done in traditional BEM books [1].

3. Transformation of domain integral to boundary by RIM

In this item, the two domain integrals appearing in Eq. (6) are transformed into equivalent boundary integrals by using the radial integration method (RIM) [11]. If the heat generation rate $Q(\boldsymbol{x})$ is a known function of the coordinates $\boldsymbol{x}$, RIM can be directly employed to transform the first domain integral in Eq. (2) to the boundary referring to Ref. [12]. However, for the last domain integral of Eq. (6), since the temperature is unknown, the formulations of RIM cannot be directly used. In order to solve this problem, the unknowns are approximated by series of prescribed radial basis functions (RBFs). In this paper, two different approaches based on the radial basis functions are implemented. Select $\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})$ and $T(\mathbf{x})$ as radial basis functions and named BEM1 and BEM2, respectively.

3.1. $\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})$ as radial basis functions

For the domain integral appeared in Eq. (6), the normalized temperature $\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})$ can be expressed as

$$\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})=\sum_A \alpha^{iA}\phi^A(R)+a^{ik}x_k+a^{i0} \tag{9a}$$

$$\sum_A \alpha^{iA}=\sum_A \alpha^{iA}x_j^A=0 \tag{9b}$$

where, $R=\|\mathbf{x}-\mathbf{x}^A\|$ is the distance from the application point A to the field point $\mathbf{x}$, α^{iA} and a^{ik} are coefficients to be decided. Usually, the application point A includes all boundary nodes and some internal points. The commonly used radial basis function $\phi^A(R)$ can be found in Refs. [12–15]. In this paper, the following 4th order spline-type RBF is used:

$$\phi^A(R/d_A)=\begin{cases}1-6\left(\frac{R}{d_A}\right)^2+8\left(\frac{R}{d_A}\right)^3-3\left(\frac{R}{d_A}\right)^4 & 0\le R\le d_A \\ 0 & R\ge d_A\end{cases} \tag{10}$$

where d_A is the support size for the application point A. The coefficients can be defined by applying the application point A in Eq. (9) at each node, which results in the following matrix equation:

$$\left\{\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})\right\}=[\phi]\{\alpha^i\} \tag{11}$$

where, $\{\alpha^i\}$ is a vector consisting of the coefficient α^{iA} for all application nodes and a^{ik}. If no two nodes share the same coordinates, the matrix is invertible. Let $\frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})=\frac{\partial k(T(\mathbf{x}))}{\partial T(\mathbf{x})}T(\mathbf{x})\frac{\partial T(\mathbf{x})}{\partial x_i}$, then $\{\alpha^i\}$ can be expressed as

$$\{\alpha^i\}=[\phi]^{-1}\left\{\frac{\partial k(T(\mathbf{x}))}{\partial T(\mathbf{x})}T(\mathbf{x})\frac{\partial T(\mathbf{x})}{\partial x_i}\right\} \tag{12}$$

where, $\frac{\partial T(\mathbf{x})}{\partial x_i}$ is unknown, in the same manner as above, Let

$$T(\mathbf{x})=\sum_A \beta^A\phi^A(R)+\beta^k x_k+\beta_0 \tag{13}$$

$$\sum_A \beta^{iA}=\sum_A \beta^{iA}x_j^A=0 \tag{14}$$