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Continuous–discontinuous hybrid boundary node method for frictional contact problems

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ABSTRACT

This paper presents a continuous–discontinuous hybrid boundary node method for frictional contact problems. In this method, the outer and internal boundaries are divided into several individual segments, for a continuous segment on outer boundary, the radial point interpolation method (RPIM) is employed for shape function construction, for discontinuous segments, the enriched discontinuous basis functions combined with RPIM are developed, in order to reflect the local field property of displacement and stress around crack tip, different basis functions for displacement and traction are developed for shape function construction on discontinuous segments individually. And the near tip asymptotic field functions and Heaviside function are employed for simulating the high gradient of stress field and discontinuous displacement field on contact surfaces. Besides a frictional contact theory and complementation detail for the present method is proposed, and some additional equations are developed for frictional contact iteration. Based on above technique and theory, a continuous–discontinuous hybrid boundary node method is proposed for frictional contact problems. Some numerical examples are shown that the present method is effective and can be widely used for some frictional contact engineering.

1. Introduction

The frictional contact problems are one of the most important problems in civil, mechanical and engineering areas. At the same time, it is also one of the most difficult problems to simulate by numerical method, because they are always nonlinear, and the convergence of frictional iteration is always difficult, besides, many geometric and mathematical models are also difficult to choose. Then it is a challenging task to model and simulate a frictional contact problem, such as sheet metal forming, cam mechanism rotating, bridge bearing, impact and penetration. Those types of problems can be modeled with adaptive finite element method (FEM) method using the penalty method [1–2] and variational inequalities [3], but high-level mesh density must be enforced around contact region. Hughes et al. [4] proposed a finite element analysis method for contact problem with a small deformation assumption.

Besides widely used of FEM for frictional contact problems, boundary element method (BEM) is also employed for simulating frictional contact problem, such as, Xiao and Yue [5] proposed a generalized Kelvin solution based BEM for contact problems of elastic indenter on functionally graded materials, and Gun and Gao [6] proposed a quadratic BEM formulation for continuous non-homogeneous, isotropic and linear elastic functionally graded material contact problems with friction. Olukoko et al. [7] gave a review of three alternative approaches to modeling frictional contact problems using the boundary element method. Hack and Becker [8] developed a local axes boundary element formulation for analysis of frictional contact problems under tangential loading. Gun [9] used boundary element method to simulate 3D elasto-plastic contact problems with friction.

Many meshless methods have been proposed in literature for analysis frictional contact problems. Aimed to overcome contact constraints and solving contact problems, an adaptive meshless method (MLM) for solving mechanical contact problems is proposed by Li and Lee [10], which automatically insert additional nodes into large error regions identified in terms of mechanical stresses, and it is a combination of a sliding line algorithm and the penalty method, thus, it can solve nonlinear contact problems with large deformation. Belaasilia et al. [11] applied a numerical mesh-free model for simulating elasto-plastic structures with contact, which is based on the asymptotic numerical method and it is in the meshless collocation framework. Gun [12] presented a quadratic meshless boundary element formulation for isotropic damage analysis of contact problems with friction.

As a widely used meshless method, the meshless local Petrov–Galerkin (MLPG) approach for large deformation contact analysis is developed by Hu et al. [13], in which the MLPG approach was based on a local weak form with RBF coupled with polynomial basis function. Xiao

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et al. [14] combined subdomain variational inequality, MLPG method and radial basis functions for solving two-dimensional contact problems, which is based on the Heaviside step function and radial basis functions.

A new complex variable element free Galerkin (EFG) was investigated by Li et al. [15], in which the technique based on Galerkin weak form and the penalty method was used to impose essential boundary conditions. Timesli et al. [16,17] proposed a new algorithm on moving least square method to simulate material mixing in friction stir welding, and furthermore, Mesmoudi et al. [18] developed a 2D mechanical-thermal coupled model to simulate material mixing observed in friction stir welding process. Belytschko and Fleming [19] proposed a contact algorithm based on the penalty method combined with EFG method for contact problems. Tiago and Pimenta [20] implemented element free Galerkin with moving least square (MLS) to nonlinear analysis of plates undergoing arbitrary large deformations. Ullah and Augarde [21] developed an adaptive meshless approach based on EFG method for nonlinear solid mechanics. EFG was applied in the simulation of forging process by Guedes and Cesarde Sa [22], in which blending finite elements with EFG in order to overcome the difficulty of meshless methods in dealing with essential boundary conditions.

Besides, Youssef et al. [23] proposed a numerical mesh-free model for simulating elasto-plastic structures with contact, which was based on the asymptotic numerical method. Chen et al. [24] discussed some recent enhancements in meshfree methods for incompressible boundary value problems, in which a mixed transformation method and a boundary singular kernel method were applied for imposition of essential boundary condition and contact constraints. Campbell et al. [25] proposed a contact algorithm for smoothed particle hydrodynamics (SPH), Attaway et al. [26] coupled SPH and finite elements for a contact problem, Kulasegaram et al. [27] proposed a new approach to handle the contact between Lagrangian SPH particles and rigid solid boundaries. Kim et al. [28] used a meshless method for a frictional contact problem with a rigid body, in which a continuum-based shape design sensitivity formulation was proposed. Chen et al. [29] used a meshless method based on the reproducing kernel particle method (RKPM) for metal forming analysis. Li et al. [30,31] employed radial point interpolation method for contact analysis of solids and metal forming process. The boundary singular kernel method was proposed by Chen and Wang [32] for computation of contact problems.

Hybrid boundary node method (HBNM) [33] is derived from boundary node method, which is first proposed by Mukherjee [34], short after that, singular hybrid boundary node method is proposed by Miao et al. [35]. Later, Yan et al. employed dual reciprocity method (DRM) [36] to HBNM, and proposed the dual reciprocity hybrid boundary node method (DHBNM) to solve inhomogeneous [37], dynamic [38], nonlinear [39], and convection-diffusion problems [40] etc. furthermore, based on Shepard interpolation method and Taylor expansion, Yan et al. [41] propose a new shape function constructing method, i.e., the Shepard and Taylor interpolation method (STIM), and develop a continuous–discontinuous hybrid boundary node method [42] for crack propagation.

In this paper, a continuous–discontinuous hybrid boundary node method for frictional contact problems is presented, in which the outer and internal boundaries are divided into several individual segments, for continuous segments on outer boundary, the radial point interpolation method (RPIM) is employed for shape function construction, for discontinuous segments, the enriched discontinuous basis functions combined with RPIM are developed, and different basis functions are employed to construct shape functions for displacement and traction respectively to reflect the local field property of displacement and stress around crack tip. And the near tip asymptotic field functions and Heaviside function are employed for simulating the high gradient of stress field and discontinuous displacement field on contact surfaces. Besides a frictional contact theory and complementation detail for the present method is proposed, and some additional equations are developed for frictional contact iteration. Based on above technique and theory, a continuous–discontinuous hybrid boundary node method is proposed for frictional contact problems. Some numerical examples are shown that the present method is effective and can be widely used for some frictional contact engineering.

2. Continuous–discontinuous hybrid boundary node method

Consider a calculating domain Ω, and its boundary is Γ. According to theory of hybrid boundary node method, only boundary needs to be discrete, and the boundary is divided into several continuous segments in this method, and the variable interpolation is approximated on each continuous segment individually, then the shape function on continuous segment is constructed by radial point interpolation method, and discontinuity is approximated by the enriched discontinuous interpolation.

2.1. Continuous interpolation

Radial point interpolation method is employed in this section to construct shape function for the common continuous segments, by which the shape function has the delta function property, and boundary conditions can be applied easily and directly, and computational expense can be greatly reduced compared to moving least square.

According to theory of RPIM, the variables of displacement u on a continuous segment can be expressed as [43]

$$u(s) \approx u^h(s) = \sum_{i=1}^{N_S} R_i(r) a_i + \sum_{j=1}^{m} P_j(s) b_j \tag{1}$$

where N_S is the node number on approximating segment, s denotes parameter coordinate of approximating segment; and m ($m < N_S$) is the number of monomials basis, and the monomials basis is $P_j(s) = s^k \quad k = 0, 1, \ldots, m$; a_i, b_j are approximating coefficient; $R_i(r)$ is the RBF, for example: multi-quadrics (MQ) $R_i(r) = (r_i^2 + c^2)^q$; Gaussian (EXP) $R_i(r) = \exp[-b r_i^2]$; thin plate spline (TPS) $R_i(r) = r_i^n \ln(r_i)$.

In order to obtain the constants coefficient a_i and b_j, Eq. (1) is enforced to be satisfied at N_S nodes at approximating segment, and combined with a constraint of the monomial basis and constant coefficient a_i, one get the system equations as follows [43]:

$$\tilde{\mathbf{u}}_0 = \begin{Bmatrix} \mathbf{u}_0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{P}_0 \\ \mathbf{P}_0^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \mathbf{G}\mathbf{a}_0 \tag{2}$$

where $\mathbf{a}_0^T = [a_1, a_2, \ldots, a_{N_S}, b_1, b_2 \ldots b_m]$, $\mathbf{u}_0^T = [u(s_1), u(s_2), \ldots, u(s_{N_S}), 0, 0, 0]$, and $\mathbf{R}_0$ is the matrix of RBF values on each approximating nodes, $\mathbf{P}_0$ is the matrix of the monomials basis values on each approximating nodes.

Solving Eq. (2), one can get the approximating coefficients, which is

$$\mathbf{a}_0 = \mathbf{G}^{-1} \tilde{\mathbf{u}}_0 \tag{3}$$

Substituting Eq. (3) into Eq. (1), one can get the shape function of the present method, which can be given as

$$\mathbf{\Phi}^T(\mathbf{s}) = [\Phi_1(s), \Phi_2(s), \ldots, \Phi_{N_S}(s), \ldots, \Phi_{N_S+m}(s)] = [\mathbf{R}^T(\mathbf{r}) \quad \mathbf{P}^T(\mathbf{s})] \mathbf{G}^{-1} \tag{4}$$

So variables for the displacement on the common continuous boundary can be obtained as [43]

$$\tilde{\mathbf{u}}(\mathbf{s}) = \mathbf{\Phi}^T(\mathbf{s}) \mathbf{u} = \begin{Bmatrix} \tilde{u}(s_1) \\ \tilde{u}(s_2) \\ \vdots \\ \tilde{u}(s_{N_S}) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{N_S} \Phi_i(s_1) u_i \\ \sum_{i=1}^{N_S} \Phi_i(s_2) u_i \\ \vdots \\ \sum_{i=1}^{N_S} \Phi_i(s_{N_S}) u_i \end{Bmatrix} \tag{5}$$

in which $\mathbf{u}^T = [u_1, u_2, \ldots, u_{N_S}]$. The same as Eq. (5), variables for the boundary traction can also be approximated by this shape function, which is

$$\tilde{\mathbf{t}}(\mathbf{s}) = \mathbf{\Phi}^T(\mathbf{s}) \mathbf{t} \tag{6}$$

in which $\mathbf{t}^T = [t_1, t_2, \ldots, t_{N_S}]$ are nodal values on approximating nodes on continuous segment.