Engineering Analysis with Boundary Elements 87 (2018) 47–65

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A boundary element formulation to perform elastic analysis of heterogeneous microstructures

Gabriela R. Fernandes[b,*], Guilherme A. Ohland[a], Jordana F. Vieira[b]

[a] FEIS – Faculdade de Engenharia de Ilha Solteira, CEP 15385-000, Av. Brasil Sul, 56 - Centro, Ilha Solteira - SP, Brazil
[b] Civil Engineering Department, Federal University of Goiás (UFG) – Regional Catalão, Av. Dr. Lamartine Pinto de Avelar, 1120, Setor Universitário, CEP 75700-000 Catalão, GO, Brazil

A R T I C L E I N F O

Keywords:
Boundary elements
Stretching problem
Zoned plates
Homogenization techniques
Multi-scale analysis
RVE

A B S T R A C T

A BEM formulation to perform elastic analysis of heterogeneous microstructures is proposed in the context of multi-scale analysis. The microstructure, also denoted as RVE (Representative Volume Element), is modeled as a zoned plate where voids or inclusions can be considered inside a matrix. Thus, each sub-region represents either the matrix or an inclusion, where different values of Poisson's ratio and Young's modulus can be defined. The RVE equilibrium equation is solved in terms of displacement fluctuations according to the formulation proposed in de Souza Neto and Feijóo (2006). Only elastic behavior is considered for matrix and inclusions, although the proposed model can be extended to consider dissipative phenomena. To make the micro-to-macro transition necessary in a multi-scale analysis, the homogenized values for stress and constitutive tensor have to be computed adopting homogenization techniques. Some numerical examples of heterogeneous microstructures are presented and compared to a FEM model to show the accuracy of the proposed model.

1. Introduction

In general, the materials, even the metallic, are heterogeneous at the micro and grain scale. The concrete, as example, has a very complex microstructure, where the mortar and aggregates present different elastic properties and non-linear behavior. Moreover, the pre-existence of initial defects in the material microstructure as micro-cracks and micro-voids can also affect considerably the structure stiffness. The material microstructure can be also appropriately manipulated by adding certain constituents to a matrix phase, in order to change the material properties to attend specific applications, as the MMCs (Metal Matrix Composites). As any heterogeneity of the material as well as the micro-cracking initiation and propagation in the micro-scale affect directly the macro-continuum response, modeling heterogeneous material in different scales is very important to better represent the behavior of such complex materials [1–8]. In many situations the traditional phenomenological approach for constitutive description does not provide a sufficiently general predictive modeling capability.

In the multi-scale modeling considered in this work, the RVE (Representative Volume Element) represents the microstructure, at grain level, of the macro-continuum at the infinitesimal material neighborhood of a point. Thus, usually a RVE must be associated to every point of the macro-continuum where the stresses and the constitutive tensor have to be computed, although there are some works that only define RVEs in the region where the stresses are higher. To make the transitions from macro-continuum to micro-continuum as well as from micro-continuum to macro-continuum homogenization techniques have to be applied in order to define the strain and stress vectors as well as the constitutive tensor related to the macro-continuum in terms of their respective fields in the RVE (see [9–15]). In most of the works about multi-scale analysis the Finite Element Method (FEM) is used to model both the macro and micro continuums [16–28]. Recently, some formulations adopting the Boundary Element Method (BEM) to model one or even both scales have been proposed (see [29–33]). In [31,32] the FEM formulation proposed in [11] to model the RVE has been coupled to the BEM formulations proposed in [34] to perform multi-scale analysis of plates considering either the simple bending or the stretching problem.

The boundary element method (BEM) has already proved to be a suitable numerical tool to deal with plate problems (either bending or stretching). It is particularly recommended to compute the effects of concentrated (in fact loads distributed over small areas) and line loads, as well to evaluate high gradient values as bending and twisting moments, and shear forces. Moreover, the same order of error is expected when computing displacements and forces, because the tractions are not obtained by differentiating approximation functions as for other numerical techniques. In this context, it is worth mentioning three edited

* Corresponding author.
E-mail address: gabrielar.fernandes@ufg.br (G.R. Fernandes).

https://doi.org/10.1016/j.enganabound.2017.11.006
Received 23 January 2017; Received in revised form 1 November 2017; Accepted 13 November 2017
0955-7997/

books [35–37] containing BEM formulations showing several important applications in the engineering context.

Note that for microstructures whose homogenized constitutive behavior is stable, which is the case considered in this work, the RVE homogenized response is objective, i.e., it does not depend on the RVE size. Thus, in this case, the existence of a RVE is a well-accepted concept (see [5,10,11,38]). On the other hand, when the heterogeneous microstructure is composed by materials presenting softening behavior which can lead to failure phenomena, the RVE existence has been discussed in several works, because using classical homogenization techniques the RVE homogenized response can lose its objectivity (see [7,39–41]).

In this paper a BEM formulation to perform elastic analysis of heterogeneous microstructures is proposed in the context of a multi-scale analysis, where the microstructure is represented by the RVE (Representative Volume Element). Note that the proposed BEM formulation could be also used to model the stretching problem of plates composed by different materials, but this case will not be considered in this work. In the proposed model the RVE is considered as a zoned plate, where each subregion defines either the matrix or an inclusion. As this study is made in the context of a multi-scale analysis, no tractions on the RVE boundary or loads inside its domain will be prescribed, i.e. only in-plane displacements, computed from a constant strain tensor, will be prescribed to the boundary nodes. In a multi-scale analysis, this constant strain tensor imposed to the RVE boundary refers to the strain tensor of the macro-continuum point represented by the RVE which is computed according to the numerical model adopted to solve the macro-continuum problem. After solving the RVE equilibrium problem in a multi-scale analysis, the homogenized values for the stress vector and the constitutive tensor have to be computed in order to check the equilibrium equation defined for the macro-continuum. Thus, to validate the proposed formulation, these homogenized values will be compared to the ones obtained by a FEM formulation.

Several works have already considered this sub-region technique to model stiffened plates or plates composed by different materials (see [42–47]). In the model presented in [44], as example, a BEM formulation for the coupled stretching-bending analysis of building floor structures is developed, where the building floor is modeled by a zoned plate where each sub-region defines a beam or a slab. The beams are modeled as narrow sub-regions with larger thickness, being all sub-regions represented by a same reference surface, so that the eccentricity effects are taken into account. Besides, the beams and slabs can be composed by different materials, i.e., the sub-regions can be defined with different values of Poisson's ration and Young's modulus. The integral equation for in-plane displacements presented in [44] without considering the bending effects is similar to the one presented in this paper, although in [44] no voids could be considered. Moreover, in the proposed model the inclusions boundaries are not coincident to the external boundary, as happened in the previous model for the external beams.

In the proposed model, to define the RVE equilibrium problem, we assume that its displacement field is divided into two parts: one obtained from the constant strain imposed by the macro-continuum to the RVE boundary and another one denoted as displacement fluctuation, according to the formulation proposed in [11]. Note that there is always a displacement fluctuation field when the strain field over the RVE is not homogeneous, i.e., even when no dissipative phenomenon is taken into account over the RVE there will be a displacement fluctuation field if the RVE is heterogeneous (composed by different materials). In the proposed formulation the RVE equilibrium equation, as well as the equations to compute the homogenized stress and constitutive tensors are the ones proposed in [11], where the RVE equilibrium problem is solved in terms of displacement fluctuations. Note that despite we use the same equilibrium equation and the same equation to compute the homogenized stresses the numerical results are not exactly the same, as the normal forces inside the domain as well as the tractions along the boundary are computed according to each numerical model (BEM or FEM). In the proposed model the microstructure (RVE)

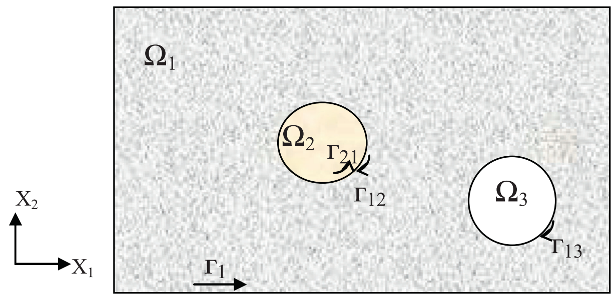

Fig. 1. Heterogeneous microstructure represented by a zoned plate.

is considered as a zoned plate where inclusions or voids are defined inside the matrix and different Young's modulus and Poisson's ration can be defined for inclusions and matrix. In this work only elastic behavior can be adopted for matrix and inclusions, but in a future work the proposed formulation will be extended to consider dissipative phenomena over the RVE. Besides, after validating the proposed model considering dissipative phenomena, cohesive-contact finite elements can also be considered in the formulation (see [48]) to better evaluate the fracture process. Then multi-scale analysis could be performed, considering only BEM, after coupling this extended formulation to other BEM formulation to model the macro-continuum, as example the ones developed in [34]. Note that in [48] the FEM formulation [11] to model the RVE has been considered where besides the triangular finite elements used to model the RVE domain, cohesive-contact finite elements have been defined at interfaces between the matrix and inclusions in order to model the phase debonding. The accuracy of the proposed model is confirmed by numerical examples whose results have been compared to the Finite Element formulation presented in the works [11,31,32].

2. Basic equations

In this section the basic equations and values related to the plate stretching problem are defined, which represents the problem to be studied in the microstructure denoted as RVE (Representative Volume Element). In general, the domain Ω of a heterogeneous microstructure is assumed to consist of a solid part, Ω^s and a void part Ω^v, being $\Omega = \Omega^v \cup \Omega^s$, where the solid part can be made of distinct materials (or phases), each one defined by a sub-domain, whose material can have different elastic properties. For simplicity, in this work we shall consider only microstructure whose void or inclusions parts do not intersect the external boundary, i.e., it is assumed that the microstructure external boundary is given by Γ_1 (see Fig. 1). Without loss of generality, let us consider the microstructure depicted in Fig. 1 represented by a zoned plate, where sub-region Ω_1 represents the matrix whose external boundary is Γ_1, sub-region Ω_2 is an inclusion and Ω_3 represents a void. Besides, Γ_{jk} represents the interface between the adjacent sub-regions Ω_j and Ω_k, being the Cartesian system of co-ordinates (axes X_1 and X_2) defined on the plate surface, as shown in Fig. 1.

For a point placed at any of those plate sub-regions, the following in-plane equilibrium equation can be defined:

$$N_{ij},_j + \bar{b}_i = 0 \quad i, j = 1, 2 \tag{1}$$

where $\bar{b}_i = b_i t$, being t the plate thickness and b_i the body forces distributed over the plate middle surface and N_{ij} is the membrane internal force, which, for plane stress conditions, can be written in terms of in-plane deformations ε_{ij} as follows:

$$N_{ij} = \frac{\overline{E}}{\left(1 - \nu^2\right)} \left[\nu \; \varepsilon_{kk} \delta_{ij} + (1 - \nu) \varepsilon_{ij} \right] \tag{2}$$

where $\bar{E} = Et$, being E the Young's modulus, ν the Poisson's ratio and δ_{ij} the Kronecker delta.