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Phase field simulation of Rayleigh–Taylor instability with a meshless method

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ABSTRACT

The purpose of this paper is a numerical study of Rayleigh–Taylor instability problem in two dimensions, based on phase-field (PF) formulation and diffuse approximate method (DAM) meshless solution procedure, enabling single-domain fixed-node approach for coping with moving boundary problems. The problem is formulated based on three physically different models that reduce to solving Cahn–Hilliard equation in addition to the Navier–Stokes equations for incompressible fluids. The governing equations are solved by using explicit time discretization. DAM is structured with second order polynomial basis, Gaussian weighting, upwinding and local domain support. The pressure–velocity coupling is performed by the fractional step method. The assessment of the method is carried out based on different node density, weighting, and the size of the local domain support. The novel approach is verified by reproducing the boundary dynamics, consistent with the previously published results. A detailed comparison with the volume of fluid finite volume approach is presented. The combination of PF and DAM provides a valuable numerical tool for solving immiscible convective hydrodynamics problems. The paper represents a pioneering attempt in solution of Rayleigh–Taylor instability problem by a meshless solution of the phase-field formulation of the problem.

1. Introduction

The fluid interface becomes unstable when a heavier fluid is placed over a lighter fluid in a gravitational field. A perturbation of this interface has a tendency to increase with time, producing a phenomenon known as Rayleigh–Taylor (RT) instability. This phenomenon describes the entrance of the fluid with a higher density into the fluid with lower density in the form of mushroom-shaped protrusions. The RT instability phenomena was initially discovered by Rayleigh [1] and after that applied to explanation of all accelerated fluids by Taylor [2]. This instability has also been used to describe a wide range of problems, such as inertial confinement fusion [3], supernova explosions and remnants [4,5], nuclear weapon explosions [6], oceanography [7], and atmospheric physics [8], and was for the first time numerically implemented in the 1970s [9].

The dynamic variables required to describe the motion of fluids are the velocity and the pressure, which are highly sensitive to the density and the viscosity. Boussinesq approximation [10] is typically applied for buoyancy-driven flows with small density variations. It has successfully been employed to obtain the numerical solution of RT instability using Lagrangian–Eulerian vortex method [11] and a new model proposed for the development of the RT instability in the Boussinesq limit, using concentrations of vorticity along the interface [12]. Numerical simulation of miscible RT instability in both two and three dimensions with the Boussinesq approximation using spectral element and pseudo spectral method has been carried out in [13] and the dynamics of RT instability of inviscid and viscous fluids has been analysed in [14].

One of the major difficulties in the study of free and moving boundary problems is the presence of the interface, which may undergo severe topological deformations, such as breakup and merging. A straightforward way to handle the moving interface is to employ the moving mesh that has the grid points on the interface, which deforms as determined by the flow on both sides of the boundary. For this purpose the recently developed boundary meshless methods [15,16] turn out to be particularly suitable. Unfortunately, these methods break down when large displacement of internal domains causes mesh entanglement or when

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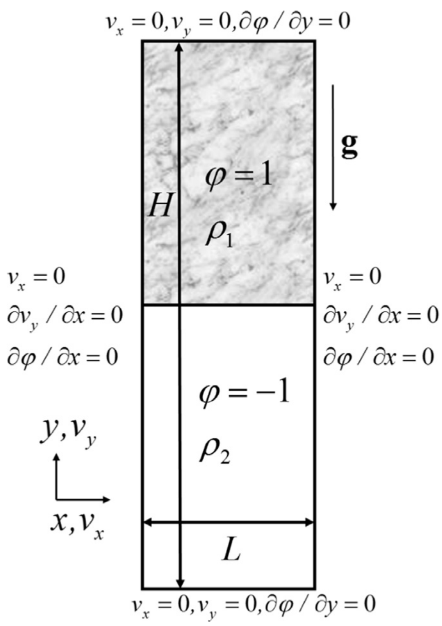

Fig. 1. Scheme of the geometry, initial conditions and the boundary conditions of the Rayleigh–Taylor instability problem.

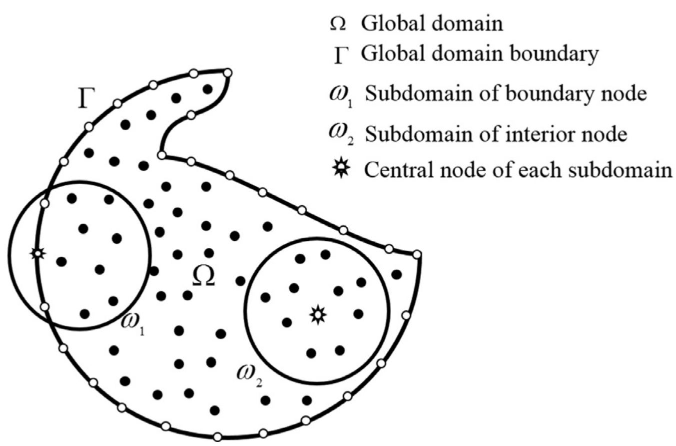

Fig. 2. Scheme of the discretization with illustration of the subdomains with boundary ω_1 and domain ω_2 computational nodes.

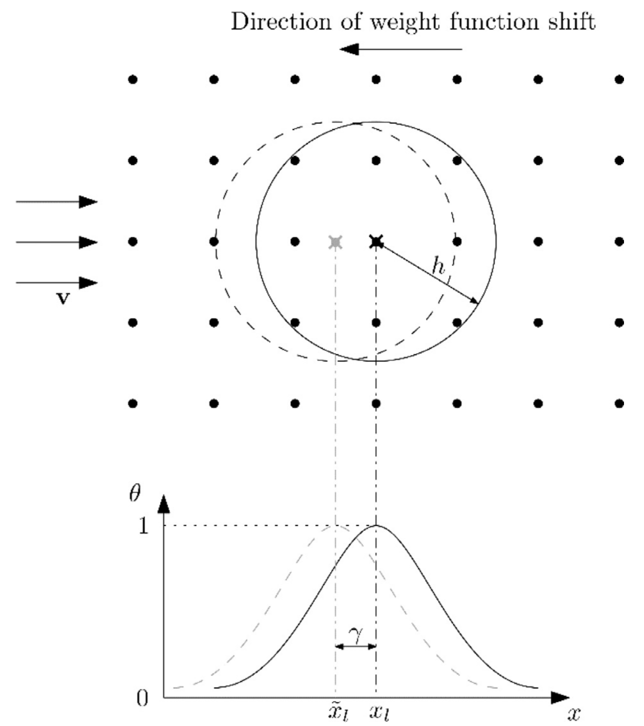

Fig. 3. Schematic of central and upwind Gaussian weight function. The solid-line circle represents the influence domain of the central node and the dashed-line circle is the influence domain of the shifted central node.

the interfaces undergo singular topological changes. In order to overcome this difficulty, fixed grid methods such as Volume of Fluid (VOF) [17] and level set method [18] have been developed to regularize the interfaces. These methods convert the Lagrangian description of a geometric motion into Eulerian description and, instead of computing the flow of two fluids with appropriate boundary conditions on the interface, represent the interfacial tension as a body force or bulk stress spread over a narrow region covering the interface. So, a single set of governing equations can be solved over the entire domain using fixed grid in an Eulerian framework.

Phase Field Method (PFM) [19] is a fixed grid technique, differing from the aforementioned methods by assuming that the interface is diffuse in a physical rather than in a numerical sense. It provides a useful tool for capturing the evolution of complex interfaces and treating the topological changes of the interface. In the PFM, an interface is described as a finite volumetric zone across which the physical properties (density, viscosity, phase field variable, etc.) vary steeply and continuously. The shape of the interface is determined by minimizing the free energy of the system [20] and no explicit interphase boundary condition is required at the moving boundary. The surface tension appears as a surface free energy per unit area caused by the gradient of the phase field variable.

In the Level Set Method (LSM), which also belongs to the fixed grid technique approaches, the interface is defined by a level set function, initialized as a signed distance function from the interface, positive on one side and negative on other side of the interface. The interface is represented by the zero level of the level set function. The main drawback of LSM is that the fronts evolve as a solution of transport equation for level set function, which causes the level set function to lose the distance function properties at later times based on the calculated velocity fields. This appears as smearing of the interface and causes difficulty to ensure mass conservation. In VOF method, the interface needs to be reconstructed from the discontinuous fraction function after each time step, which is a computationally demanding task. In contrast, PFM constructs the interface by taking the gradient of the chemical potential into account, so the effect of surface tension on flow fields can be treated without complicated topological calculations of the interfacial profile. In addition, PFM does not require conventional algorithms like Donor–Acceptor [17] and Flux Line-segment Advection and Interface Reconstruction (FLAIR) [21], for reconstruction and advection of an interface.

In the literature, many applications of the PFM to the multiphase flows can be found. A phase field model has been developed by using the Boussinesq approach to simulate the three-phase flows [22] and a semi discrete Fourier-spectral method has been used to approximate the phase field model based on the Boussinesq approximation for the mixture of two incompressible fluids [23]. The phase field model with variable density and viscosity has also been used to simulate the incompressible two phase system in [24]. Various surface tension implementations in PFMs have been elaborated in [25]. Many numerical methods including boundary integral methods [26], front tracking methods [27], VOF method [28], and LSM [29] have been used to analyse the RT instability. Additionally, the dynamics of RT instability of two immiscible fluids in the limit of small Atwood numbers together with surface tension effect has been numerically analyzed using PFM [30]. In [31], a