Stability analysis of the thin plates with arbitrary shapes subjected to non-uniform stress fields using boundary element and radial integration methods

L. Najarzadeh, B. Movahedian, M. Azhari*

Department of Civil Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

ARTICLE INFO

ABSTRACT

In this paper, a boundary element method is applied to buckling analysis of thin plates with arbitrary shapes under various load types. The governing differential equation is converted into equivalent integral equation using the static fundamental solutions of the biharmonic equation. The arising domain integrals due to in-plane stresses are evaluated applying the radial integration method. The in-plane stresses are implemented through Gaussian integration points along radial direction in the convex domains. For the concave domains, an idea has been introduced to compute radial integration along new direction from an auxiliary point to the field point. This method can be easily applied to buckling analysis of thin plates with arbitrarily distributed and concentrated edge loading. Six sample problems are presented to illustrate the effectiveness and accuracy of the proposed method.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

In modern engineering design of steel structures, attentions are paid to the more optimized and economic components of structures by eliminating the overdesigned members. Hereupon, a comprehension of plate stability subjected to compressive loads is of great importance. These loads can develop either uniform or non-uniform in-plane stresses.

Buckling analysis of plates has been the main subject of many studies. The presented results of most analytical methods are limited to rectangular plates based on simplifying the stress distribution. However, several numerical methods have been introduced to cope with the buckling problem of plates with complex geometries and loading conditions. Among recent studies, the buckling problem of rectangular plates under locally distributed edge forces was solved using differential quadrature method [1]. Mijušković et al. implemented an analytical solution for stability analysis of rectangular plates with different boundary conditions for different loading based on exact stress functions implementation through Ritz energy technique [2,3].

The Boundary element method as an efficient and powerful numerical solution has been widely utilized to treat the stability analysis of plates [4–6]. The main feature of boundary element method is that the discretization is limited to the boundary. However, in the solution of buckling problem using the fundamental solution of bending analysis, domain integrals are emerged in the integral equations. These domain integrals have been treated using meshless or mesh dependent tech-

niques so far. A wide range of studies have been done to tackle such integrals using different domain discretization techniques. In mesh dependent studies, the discretization is the same as finite element method, using continuous and discontinuous triangular or rectangular cells [1,6,7]. Nevertheless, since cell-integration scheme eliminates the pure boundary feature of boundary element method, attempts were made to convert the domain integrals into boundary ones. The dual reciprocity method (DRM) is one of these techniques which approximates the body force quantities by a series of prescribed basis functions. It converts the domain integrals to the boundary using particular solutions derived from the differential operator of the problem with these basis functions. This method has been used by Tsiatas and Yiotis for the treatment of domain integrals in the solution of buckling and vibration of orthotropic plates [5]. Another efficient and simple tool of transformation is the radial integration method (RIM). This method was first proposed by Gao and it was able to convert any kind of domain integrals to the boundary without the need for the particular solution [8]. Besides, strong and weak singularities were removed by transforming the domain integrals to the boundary. The radial integration method has been used by other researchers for different plate problems [9–11]. It was employed by Aliabadi et al. for the stability analysis of isotropic and anisotropic plates under uniform loading [12]. Efforts were given to the stability analysis of plates with different geometry and loading conditions using the radial integration method in [13]. In this study, the effect of in-plane stresses was considered using a collocation method. However, this tech-

---

Q : Field Point
P_1 : Domain Source Point
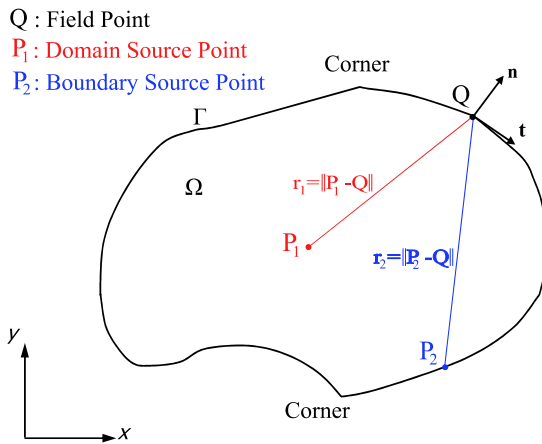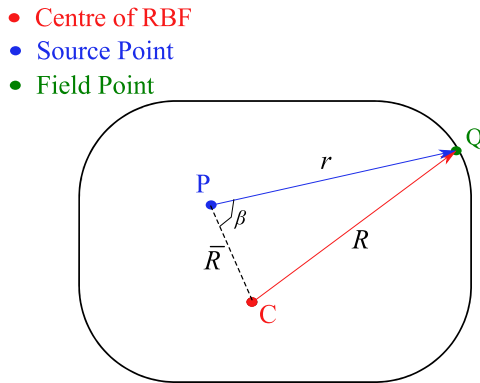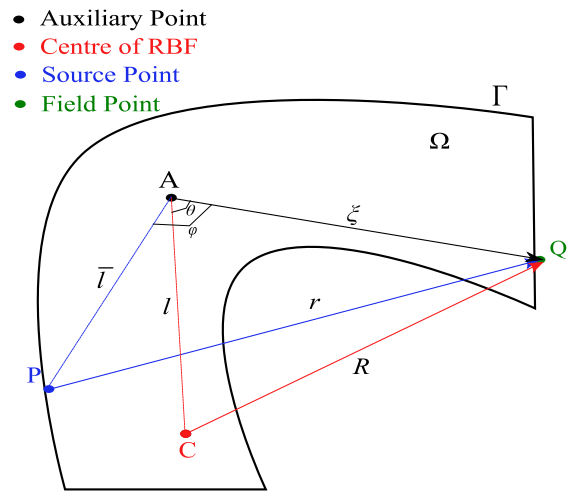P_2 : Boundary Source Point

Fig. 1. Plate geometry and notation.

Fig. 2. Schematic arrangement of source point, field point and center point of RBF to express relation between distances r and R.

Fig. 4. A sample of point arrangement in the concave domain in which r lies outside the boundaries.

nique does not seem to be suitable for the plates under forces with high stress gradient fields.

In this paper, buckling analysis of thin plates under non uniform in-plane stresses with different geometries is investigated using boundary element method in which the domain integrals are transformed to the equivalent boundary integrals using the radial integration method. The appearing in-plane stresses are established independently solving a pro-

gram for two dimensional elasticity problems. These stresses are directly implemented into the stability formulation through integration points along radial direction. Two kinds of geometries including convex and concave shapes are considered that cover various shapes. The concave geometries are dealt with introducing an auxiliary point in the transformation process. The validity and accuracy of the proposed method is investigated through the solution of different sample problems. Results are also verified with numerical and analytical analysis obtained from the literature and the finite element method.

The layout of the paper is as follows. In Sections 2 and 3, the governing differential and integral equations of the thin plate are stated, respectively. Section 4 includes the numerical solution in which the transformation process is presented for two kinds of shapes, convex and concave, within two subsections. The boundary element discretization is introduced in Section 5, and a number of numerical examples are provided to obtain the critical load factors in Section 6. Finally, in Section 7 the conclusions of this study are summarized.
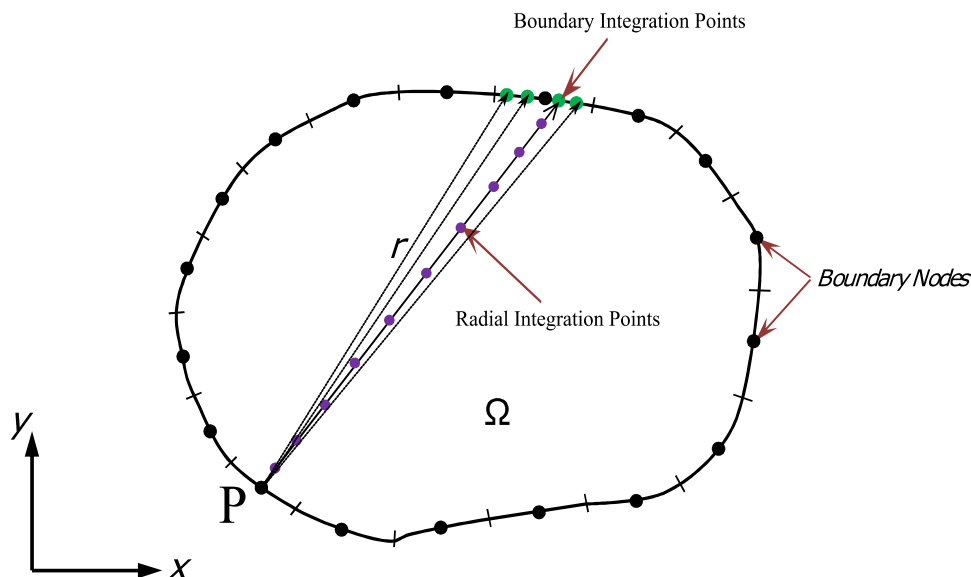
Fig. 3. Implementation of in-plane stresses through integration points along radial direction r.