Engineering Analysis with Boundary Elements 87 (2018) 153–164

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Nonlinear thermal buckling analyses of functionally graded plates by a mesh-free radial point interpolation method

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ARTICLE INFO

Keywords:
Functionally graded plate
Thermal buckling
Mesh-free radial point interpolation method
Higher-order shear deformation plate theory

ABSTRACT

This study intends to analyze nonlinear buckling behavior of functionally graded (FG) plates under thermal loading by a mesh-free method. The buckling formulation is derived based on the higher-order shear deformation plate theory in which the von Kármán large deflection assumption is employed. An improved mesh-free radial point interpolation method (RPIM) which incorporates the normalized radial basis function capable of building the shape functions without any fitting parameters is presented and utilized to scrutinize the buckling responses. The nonlinear equations are solved by the modified Newton–Raphson iterative technique. Verification of the improved RPIM is implemented by simulating several numerical examples available in the literature and comparing the outcomes with the analytical results. Detailed parametric studies demonstrate that the improved mesh-free RPIM can effectively predict the thermal buckling responses of FG plates, and the volume fraction, plate length-to-thickness ratio, aspect ratio, boundary condition have considerable effects on the critical buckling temperatures of FG plates subjected to various types of temperature variations through the thickness.

1. Introduction

Functionally graded materials (FGMs) are the advanced materials in the family of composites that have continuously varying material properties through certain spatial directions according to a predetermined rule [1]. Owing to their exceptional material properties compared to the conventional laminated composites, FGMs have been used for the structural components associated with high temperatures and large temperature gradients such as spacecraft heat shield, heat exchanger tubes, fusion reactors and semiconductors [2]. FGMs are microscopically inhomogeneous composites and typically consist of two microstructural phases such as metal and ceramic. Functionally graded (FG) plates are usually designed to optimize the desired material properties, e.g. heat and corrosion resistance by one ceramic-rich surface and mechanical strength and toughness enhancement by the other metal-rich surface.

The increasing applications of FG plates in thermal environments have expedited extensive research works on the thermal behavior. An extensive review of the thermal responses of FG plates is provided in [3]. Thermo-elastic analyses of FG plates have been conducted theoretically and numerically by many researchers based on one-dimensional constant, linear and nonlinear temperature variations and three-dimensional temperature gradients through the thickness [4–18]. Much work has also been dedicated to scrutinize the thermo-mechanical behavior of FG plates [2,19–33]. As for the thermal buckling analyses of FG plates, only a few investigations have been performed due to the complexity of the problem. Javaheri and Eslami [34] carried out buckling analyses of rectangular FG plates under four types of thermal loads. Critical buckling temperature calculated by the higher-order shear deformation theory (HSDT) model was found to be the most accurate, whist the classical laminated plate theory overestimated the buckling temperature. Ma and Wang [35] studied the nonlinear bending and postbuckling of FG circular plate by the shooting method based on the classical von Kármán plate theory. Liew et al. [36] carried out thermal buckling and postbuckling analyses for moderately thick laminated rectangular plates that contain FGMs and subjected to a uniform temperature change based on the first-order shear deformation theory (FSDT). Woo et al. [37] provided an analytical solution for the analysis of postbuckling behavior of moderately thick FG plates and shells under edge compressive loads and a temperature field by using the von Kármán theory for large transverse deflection and the higher-order shear deformation plate theory. Park and Kim [38] conducted finite element (FE) analyses of thermal postbuckling and vibration behavior of FG plates based on the FSDT. Shariat and Eslami [39] performed the thermal buckling analyses of imperfect rectangular FG plates utilizing the classical plate theory for deriving the equilibrium, stability and compatibility equations. Shen [40] discussed thermal postbuckling responses of simply supported

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https://doi.org/10.1016/j.enganabound.2017.12.001
Received 16 April 2017; Received in revised form 30 September 2017; Accepted 2 December 2017
0955-7997/

shear deformable FG plates with the initial geometric imperfection by employing the two-step perturbation technique. The governing equations were based on the HSDT that included thermal effects. Two cases of temperature field through the plate thickness, i.e. in-plane non-uniform parabolic variation and steady-state heat conduction were considered. A two-dimensional global higher-order deformation theory was presented by Matsunaga [41] for thermal buckling analyses of FG plates. Tung and Duc [42] examined buckling responses of FG plates with geometric imperfections under in-plane compressive, thermal and combined loads with the Galerkin method based on the classical laminated plate theory. The work was extended to investigate the buckling and postbuckling responses of shear deformable FG plates resting on elastic foundations based on the HSDT [43]. The effects of material and geometrical parameters and in-plane boundary restraint, foundation stiffness and imperfection were discussed. Bouazza et al. [44] established nonlinear stability equations for thick FG plates in thermal environments based on the FSDT employing power law function. Two types of thermal loading, i.e. linear temperature rise and gradient through the thickness were considered. The influence of transverse shear deformation on the critical buckling temperature was found to be significant in thick plates with large aspect ratio. Zhang and Zhou [45] explored thermal postbuckling responses of FG rectangular plates resting on elastic foundations by using the concept of physical neutral surface and the HSDT. Nonlinear approximate solutions were obtained by multi-term Ritz method for various boundary conditions. Lee et al. [46] studied the thermal buckling behavior of FG plates based on the neutral surface concept. In the formulation, linear theory of plate was adopted based on the FSDT and the steady-state thermal conduction was considered as the one dimensional heat transfer. Taczała et al. [47] presented a FE method to investigate the nonlinear stability of stiffened FG plates subjected to mechanical and thermal loads based on the FSDT.

Recently, mesh-free methods have been widely exploited to investigate numerically the responses of FG plates subjected to various types of loading. Studies include the static [48–54], dynamic [50,55–61], heat conduction [62,63] and thermo-mechanical [64–67] analyses by using diverse meshless methods based on various plate theories. On the other hand, thermal buckling analyses of FG plates by employing a mesh-free method are very scarce in the literature. Zhao et al. [68] developed the element-free *kp*-Ritz method based on the FSDT to examine the thermal and mechanical buckling behavior of FG plates. In their work, the plate bending stiffness was calculated utilizing the stabilized conforming nodal integration approach and the shear and membrane stiffness were evaluated using the direct nodal integration method to eliminate the shear locking effects of very thin plates. The method was extended to analyze the postbuckling responses of FG plates in thermal and mechanical loads [69]. Zhang et al. [70] assessed the buckling behavior of FG plates under mechanical and thermal loadings by utilizing the first-order shear deformation theory and the local Kriging meshless method based on the local Petrov–Galerkin weak-form formulation and Kriging interpolation. More information can be found in the review papers [71,72] for the modeling and analysis of FG plates and shells using a mesh-free method.

According to the literature review, it seems that thermal buckling behavior of FG plates has not been thoroughly investigated. Moreover, limited works have been conducted on the prediction of thermal buckling behavior of FG plates by employing a mesh-free method. Inspired by this, this study aims at analyzing nonlinear buckling behavior of FG plates under thermal loading by a meshless method. An improved radial point interpolation method (RPIM), in which the normalized radial basis function which enables the shape functions to be built without any supporting fixing coefficients is incorporated, based on the HSDT is presented and utilized to solve the problems. Verification of the improved RPIM is first implemented by comparing the numerical results obtained by the present method with the analytical solutions given in the literature. Detailed parametric studies are then performed to elucidate the effects of the volume fraction, plate length-to-thickness ratio,

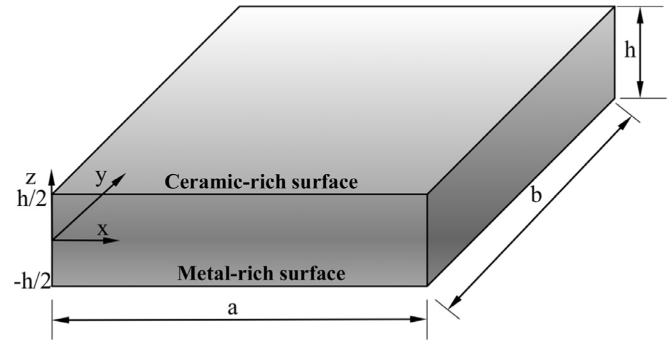

Fig. 1. Configuration of FG plate.

aspect ratio and boundary condition on the nonlinear thermal buckling of FG plates under various types of temperature variations through the thickness.

2. Formulation for thermal buckling analysis of FG plate

2.1. FG plate

FG plate consists of two kinds of materials: one is metal and the other is ceramic. Rectangular FG plate which has length a, width b and uniform thickness h with a Cartesian coordinate system (x, y, z) specified on the mid-plane $(z=0)$ is considered and illustrated in Fig. 1. The top surface of the FG plate is regarded to be ceramic rich and the bottom surface metal rich, and the region between the two surfaces is composed of the mixture of the two materials. The thermal and mechanical properties of the blended material are estimated by the rule of mixture and are assumed to vary through the thickness according to the power-law equation [21]:

$$P_e(z) = P_c V_c(z) + P_m(1 - V_c(z)) \tag{1}$$

where P_e represents the effective material properties of the FG plate such as Young's modulus E, density ρ, Poisson's ratio ν, thermal conductivity κ or thermal expansion coefficient α. P_c and P_m symbolize the material properties of the ceramic and the metal, respectively. V_c indicates the volume fraction of ceramic and is postulated to change in the z (thickness) direction only with a simple power-law distribution that takes the form [21].

$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n \quad (0 \le n \le \infty) \tag{2}$$

where n is the power-law index that dictates variation of the volume fraction V_c through the plate thickness. Note that the material is isotropic on the plane perpendicular to the z-axis. For $n=0$, it holds $P_e=P_c$ and for $n=\infty$, $P_e=P_m$. The two cases correspond to the isotropic ceramic and metal plates, respectively.

2.2. Governing equations for the FG plate

The higher-order shear deformation plate theory is adopted to describe the displacement fields, which are defined as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + f(z)\beta_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + f(z)\beta_y(x,y) \qquad \left(\frac{-h}{2} \le z \le \frac{h}{2}\right) \\ w(x,y,z) &= w(x,y), \end{aligned} \tag{3}$$

where $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ denote displacements of a point in question within the FG plate along the x, y and z directions, respectively, and $u_0(x,y)$, $v_0(x,y)$ and $w(x, y)$ are the displacements of a point on the mid-plane $(z=0)$. $\beta_x(x,y)$ and $\beta_y(x,y)$ stand for the rotations of the point with respect to the y- and x-axes, respectively. Based on the