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A node-based partly smoothed point interpolation method (NPS-PIM) for dynamic analysis of solids

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ABSTRACT

Traditional finite element method (FEM) using linear triangular element is overly-stiff and generally natural frequencies calculated would be too high. The newly developed node-based smoothed point interpolation method (NS-PIM) has overly-soft stiffness, which makes it provide natural frequencies smaller than the exact ones and has the problem of temporally instability. So this work proposed a node-based partly smoothed point interpolation method (NPS-PIM) by combing the ideas of both FEM and NS-PIM through node-based partly strain smoothing operation. Owing to the properly constructed stiffness, detailed numerical study has shown that the NPS-PIM not only successfully overcomes the temporal instability existing in the NS-PIM, but also provides more accurate results than both FEM and NS-PIM models. The use of linear triangular background cells makes the present method very competitive for solving practical engineering problems with complicated shapes.

1. Introduction

After more than half a century of development, finite element method (FEM) has become a very powerful and commonly-used technique for solving numerical solutions in science and engineering [1,2]. The FEM divides a complicated continuum into finite number of elements on the basis of Galerkin weak formulation ensuring the stability and convergence. Among the elements in the FEM of the 2D problem, the linear triangular element is the simplest one and can be generated automatically without manual operation. However, a FEM model using linear triangular elements behaves over-stiffly and the calculated solutions are generally of poor accuracy and lower convergence rate compared to other elements.

Many efforts have been made to overcome the overly-stiff problem of FEM, such as the development of hybrid FEM formulations [1] and meshfree methods have become attractive alternatives for problems in computational mechanics [1–9]. In recent years, a group of smoothed point interpolation methods (S-PIMs) have been proposed based on the generalized gradient smoothing technique for discontinues functions the newly presented G space theory, and the notion of weakened weak (W^2) formulation [1–6]. The node-based smoothed point interpolation (NS-PIM, which was originally named as linearly conforming point interpolation method) is a typical S-PIM model, which uses the point interpolation method (PIM) for creating shape functions and node-based smoothing domain for gradient smoothing operation. It was found that the NS-PIM can provide very good stress solutions, immune from the volumetric locking, work well for linear triangular mesh, and more importantly it can provide upper bound solutions in energy norm [5,11]. Therefore using the NS-PIM together with FEM, the bounds of the "exact" solution in energy norm can be obtained [5].

However due to the overly-soft property, the NS-PIM models are temporally unstable, the spurious nonezero-energy models can be found thus they fail to solve the dynamic problem directly [5,6,10,12]. Some special stabilization techniques have been developed for NS-PIM models to calculate dynamic problems [5,6]. However, the proposed measures are either using more degrees of freedom at the nodes or making the method more complicated [14,15]. Other than using the node-based gradient smoothing operation, some works for dynamic analysis have been conducted by using other types of gradient smoothing [16,17]. Motivated from the facts that NS-PIM performs over softly and FEM performs over stiffly, this work proposed a node-based partly smoothed point interpolation method (NPS-PIM) by combing FEM and NS-PIM

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through node-based partly strain smoothing operation, which is supposed to overcome the instability issue of NS-PIM and provide more accurate results for dynamic analysis.

This paper is arranged as follows. In Section 2, the idea of NPS-PIM is described. Section 3 is the standard patch test is conducted using the proposed method. Section 4 introduces the numerical examples for free and forced vibration analyses. Finally some concluding remarks are made.

2. Idea of the node-based partly smoothed point interpolation method

2.1. Briefing on the finite element method (FEM)

The discrete equation of FEM is obtained from the Galerkin weak form [1,2]

$$\int_{\Omega}(\nabla_s\delta\mathbf{u})^{\mathrm{T}}\mathbf{D}(\nabla_s\mathbf{u})d\Omega-\int_{\Omega}\delta\mathbf{u}^{\mathrm{T}}\mathbf{b}d\Omega-\int_{\Gamma_t}\delta\mathbf{u}^{\mathrm{T}}\bar{\mathbf{t}}d\Gamma=0 \tag{1}$$

where $\mathbf{D}$ is the matrix of material constants, $\mathbf{b}$ is the vector of the body forces, $\bar{\mathbf{t}}$ is the external forces vector on the natural boundary Γ_t, $\mathbf{u}$ is the trial functions, $\delta\mathbf{u}$ is the test functions, $\nabla_s\mathbf{u}$ is the gradient of the displacement field and Ω is the domain of the problem.

The displacements of the field $\mathbf{u}$ can be approximated in the form

$$\mathbf{u}^h(\mathbf{x})=\sum_{J=1}^{\mathrm{NP}}\mathbf{N}_J(\mathbf{x})\mathbf{d}_J;\quad \delta\mathbf{u}^h(\mathbf{x})=\sum_{J=1}^{\mathrm{NP}}\mathbf{N}_J(\mathbf{x})\delta\mathbf{d}_J \tag{2}$$

where $\mathbf{N}_J(\mathbf{x})$ is a matrix of shape functions, $\mathbf{d}_J$ is the vector of the nodal displacements and NP is number of nodal variables of the element.

Then substituting Eq. (2) into Eq. (1), the standard discretized system algebraic equation is obtained

$$\mathbf{K}^{\mathrm{FEM}}\mathbf{d}=\mathbf{f} \tag{3}$$

where $\mathbf{K}^{\mathrm{FEM}}$ is the system stiffness matrix and $\mathbf{f}$ is the force vector. They are assembled with the entries of

$$\mathbf{K}^{\mathrm{FEM}}_{IJ(e)}=\int_{\Omega_e}\mathbf{B}_I^{\mathrm{T}}\mathbf{D}\mathbf{B}_J d\Omega \tag{4}$$

$$\mathbf{f}_{J(e)}=\int_{\Omega_e}\mathbf{N}_J^{\mathrm{T}}(\mathbf{x})\mathbf{b}d\Omega+\int_{\Gamma_t}\mathbf{N}_J^{\mathrm{T}}(\mathbf{x})\bar{\mathbf{t}}d\Gamma \tag{5}$$

where the strain matrix is defined as

$$\mathbf{B}_J(\mathbf{x})=\nabla_s\mathbf{N}_J(\mathbf{x}) \tag{6}$$

Three nodes triangular element is used in this paper therefore the strain matrix is a constant matrix. Then Eq. (4) becomes

$$\mathbf{K}^{\mathrm{FEM}}_{IJ(e)}=\mathbf{B}_I^{\mathrm{T}}\mathbf{D}\mathbf{B}_J V_e \tag{7}$$

where V_e is area of the element.

2.2. Briefing on the node-based smoothed point interpolation method (NS-PIM)

The NS-PIM is originally proposed by Liu and Zhang et al. [10,11,13] using the generalized gradient smoothing technique [10]. The PIM shape functions are created using T-schemes which are based on triangular background cells to minimize the number of nodes and to overcome the singular moment matrix issue. The NS-PIM is a typical generalized smoothed Galerkin (GS-Galerkin) weak form method based on the normed G space theory which allows the use of discontinuous displacement functions.

In the scheme of NS-PIM, the problem domain is divided triangular background cells first and node-based smoothing domains are then formed by connecting sequentially the mid-edge points to the centroids of the surrounding triangular cells sharing node K. As shown in Fig. 1, $\Omega^{(K)}$ is the smoothing domain associated with node K and $\Gamma^{(K)}$ is the boundary of smoothing domain $\Omega^{(K)}$.

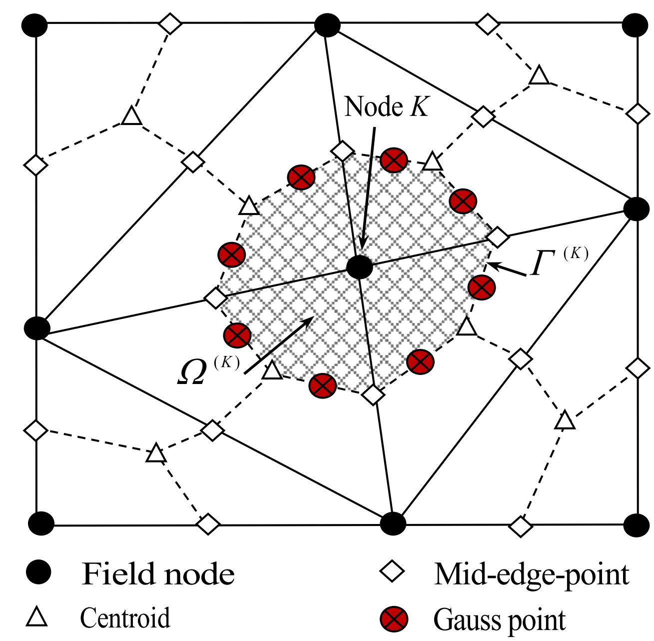

Fig. 1. Triangular background cells and node-based smoothing domains.

The discrete equation of NS-PIM is obtained from the GS-Galerkin weak form

$$\sum_{i=1}^{N_s}A_i^s\delta\bar{\boldsymbol{\varepsilon}}_i^{\mathrm{T}}(\mathbf{u})\mathbf{D}\bar{\boldsymbol{\varepsilon}}_i(\mathbf{u})-\left(\int_{\Omega}\delta\mathbf{u}^{\mathrm{T}}\mathbf{b}d\Omega+\int_{\Gamma_t}\delta\mathbf{u}^{\mathrm{T}}\bar{\mathbf{t}}d\Gamma\right)=0 \tag{8}$$

where A_i^s is the area of the smoothing domain, N_s is the number of the smoothing domains and $\bar{\boldsymbol{\varepsilon}}_i(\mathbf{u})$ is the smoothed strain obtained in the following form

$$\bar{\boldsymbol{\varepsilon}}_i(\mathbf{u})=\frac{1}{A_i^s}\int_{\Gamma_i^s}\mathbf{L}_n\mathbf{u}d\Gamma \tag{9}$$

where Γ_i^s is the boundary of the smoothing domain and $\mathbf{L}_n$ is the matrix of outwards normal components which can be expressed as

$$\mathbf{L}_n=\begin{bmatrix}n_1 & 0\\ 0 & n_2\\ n_2 & n_1\end{bmatrix} \tag{10}$$

where n_1 and n_2 are the direction cosine calculated on the boundary of the smoothing domain. The displacements of the field can be approximated in the form

$$\bar{\mathbf{u}}(\mathbf{x})=\sum_{I\in s_n}\boldsymbol{\Phi}_I(\mathbf{x})\bar{\mathbf{d}}_I=\boldsymbol{\Phi}(\mathbf{x})\bar{\mathbf{d}} \tag{11}$$

where $\boldsymbol{\Phi}_I(\mathbf{x})$ is the PIM shape function at node I, $\boldsymbol{\Phi}$ is the matrix of shape functions collecting $\boldsymbol{\Phi}_I(\mathbf{x})$, s_n is the set of nodes selected using T-schemes [12]. In this work, the cell-based T6/3 scheme was used for selecting support nodes. The specific operation is as follows: for an interior home cell, 3 nodes of the home cell and 3 remote nodes of the three neighboring cells are taken; while for a boundary home cell, only 3 nodes of the home cell are taken.

In Eq. (11), $\bar{\mathbf{d}}$ is the nodal displacements for all nodes in the entire problem domain

$$\bar{\mathbf{d}}=\left\{\bar{\mathbf{d}}_1(\mathbf{x})\quad \bar{\mathbf{d}}_2\ldots\bar{\mathbf{d}}_I\ldots\bar{\mathbf{d}}_{s_n}(\mathbf{x})\right\}^{\mathrm{T}} \tag{12}$$

Substituting Eqs. (10) and (11) into Eq. (9) the smoothed strain can be written in the following form

$$\bar{\boldsymbol{\varepsilon}}_i(\mathbf{x})=\sum_{I\in s_s}\bar{\mathbf{B}}_I(\mathbf{x})\bar{\mathbf{d}}_I=\bar{\mathbf{B}}(\mathbf{x})\bar{\mathbf{d}} \tag{13}$$