Engineering Analysis with Boundary Elements 000 (2017) 1–11

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Modelling transient heat conduction of granular materials by numerical manifold method

Jun He[a,b], Quansheng Liu[a,b], Zhijun Wu[a,b,*], Xiangyu Xu[a,b]

[a] School of Civil Engineering, Wuhan University, Wuhan 430072, China
[b] Stake Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China

ARTICLE INFO

Keywords:
Numerical manifold method
Transient heat conduction
Cohesive element method
Granular materials
Effective heat conductivity

ABSTRACT

Modelling heat conduction is of significant importance for evaluating temperature effects of granular materials. Since the randomness of the grain structure and the heat resistance characteristics of grain interfaces, to realistically modelling the heat conduction of granular materials, the heat interactive among these random-shaped grains should be correctly reflected. In this study, the numerical manifold method (NMM) is extended to model the transient heat conduction of granular materials. The random-shaped grain structure of granular materials is represented with Voronoi polygons. The heat interactive among grains is realistically simulated by inserting heat conductive cohesive elements between grain boundaries. Besides, an interfacial heat conductivity is defined for the cohesive element to better represent the heat conduct capacity of grain interfaces. As a result, the temperature jumps at grain interfaces are naturally captured due to the dual cover systems of the NMM, while the heat fluxes across the interfaces are assumed to be continuous. To validate the developed numerical method, a benchmark test is carried out. At last, effects of the grain characteristics and interfacial heat resistance on the temperature field as well as the effective heat conductivity (EHC) of a plane consists of granular material are investigated by the developed NMM.

1. Introduction

Granular materials, such as rock and concrete, are composed of random-shaped mineral grains and cemented interfaces [1]. In engineering practice, these granular materials are often subjected to different temperature conditions. Experiments have revealed that the temperature variation has significant influence on the mechanical properties of granular materials [2]. When subjected to high temperatures, the random-shaped grain geometry and the heat resistive characteristic of the grain interfaces may cause high temperature gradient, uncoordinated thermal strains and stresses in granular materials. As a result, material degradation, thermal cracking and even structure failure frequently occur. Therefore, investigating heat conduction between grains is of significant importance to successfully evaluate the thermal effects of the granular materials.

Basically, the heat conduction of granular materials can be divided into two separate processes, i.e. the heat conduction in solid grains and the heat conduction between grains. Since laboratory experiments still suffer some limitations on clearly monitoring these two processes, numerical simulations are widely adopted to solve this problem. To realistically model the heat conduction of granular materials, numerical methods that take both these two separate processes into consideration should be developed. Originally, the continuum-based numerical methods, such as the finite element method (FEM) and the extended finite element method (XFEM), are used to deal with heat conduction of granular materials [3–5]. Indeed, for relatively simple continuous geometries, accurate temperature fields and heat fluxes can be obtained by the continuum-based numerical methods. However, these numerical methods run into difficulties in realistically representing the complex grain structures and the heat interactive among grains of granular materials due to the conforming mesh requirements of the FEM [5] and the additional enrichment function requirements of the XFEM [6,7].

As powerful alternatives to overcome the limitations of the continuum-based numerical methods, some discontinuous numerical schemes have also been extensively used to investigate the heat conduction of granular materials [8–12]. By representing granular materials with bonded geometrical elements, the grain structure and heat interactive among grains can be explicitly expressed by discontinuous numerical schemes [10,12,44]. Besides, complex thermal fracturing along grain boundaries induced by heat conduction among granular materials can be successfully captured, such as the thermal fracturing predicted by the discrete element method [8,10] and the discontinuous deformation analysis [9]. Nevertheless, the discontinuous numerical schemes also

* Corresponding author at: School of Civil Engineering, Wuhan University, Wuhan 430072, China.
E-mail address: wuzh0013@e.ntu.edu.sg (Z. Wu).

https://doi.org/10.1016/j.enganabound.2017.10.011
Received 19 September 2017; Received in revised form 25 October 2017; Accepted 25 October 2017
Available online xxx
0955-7997/

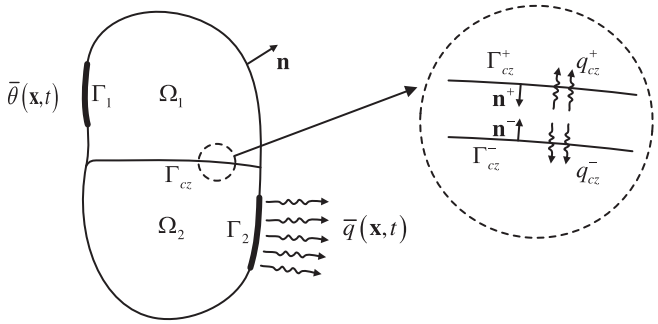

Fig. 1. Heat transfer in a granular material composed of two grains.

suffer some limitations, such as the difficulty in capturing the complex deformation and heat conduction in solid grains.

To overcome the limitations suffered by the continuous numerical schemes and the discontinuous numerical schemes, some unified continuous–discontinuous numerical methods are developed in the last two decades. One of the most widely used unified continuous–discontinuous numerical methods is the numerical manifold method (NMM) [13], which is a PU based numerical method [14]. The NMM can solve both continuous and discontinuous problems accurately. However, the main distinction between the NMM and other PU based numerical methods is that arbitrary discontinuities can be simulated by the NMM without additional mathematical treatments, which can be attributed to the dual cover systems of the NMM, i.e. the mathematical cover (MC) and the physical cover (PC) [15]. Therefore, the NMM has gained extensive attentions nowadays, especially for researchers working on complex continuous–discontinuous problems [16–28,45]. In addition, the NMM has also been extended to model thermal fracture problems [29,30].

Due to the powerful capability of the NMM in dealing with both continuous and discontinuous problems, the NMM is extended to model heat conduction of granular materials in this study. To better simulate the heat conduction of granular materials, a feasible NMM model which can reasonably represent the grain structure should be adopted. Experimental observations have found that the grain structure of granular materials [31], e.g. rock and concrete, appear more like random polygons. Therefore, the Voronoi tessellation, by which the polygonal characteristics and randomness of grain structures can be reflected, is adopted to generate the NMM model to approximate the grains of granular materials [32]. Since numerous of numerical simulations on heat conduction of continuous solids are available, numerical treatments for simulating heat conduction in solid grains by the NMM are thus straightforward. Furthermore, to more realistically capture the heat interactive among grains, the heat conductive cohesive element, which can explicitly capture the heat interactive among grains by bonding up the grain boundaries [33–38], is incorporated into the NMM platform. Besides, an interfacial heat conductivity is defined for the cohesive element to reflect the heat conduct capacities of the interfaces [38]. As a result, both the complex grain structures and heat interactive among grains of the granular materials can be explicitly considered by the developed NMM. To validate the developed method, a benchmark test is carried out. Finally, the developed NMM is applied to investigate the effects of the grain characteristics and interfacial heat resistance on the temperature field as well as the effective heat conductivity (EHC) of a plane consists of granular material.

2. Problem formulation

Without losing generality, we consider the simple transient heat conduction problem of a granular material Ω, which is composed of two separate grains Ω_1 and Ω_2 as shown in Fig. 1. The interface between the grains is termed as Γ_{cz}. It is worth noting that, the heat resistive interface Γ_{cz} will definitely influence the ultimate temperature field, which should be taken into consideration. Thus, the aim is to seek for the solution of the temperature field $\theta(\mathbf{x}, t)$ that satisfies the boundary conditions on Γ_1 and Γ_2, the interfacial heat interactive condition on Γ_{cz} and the equilibrium equation.

In absence of the internal heat source, the equilibrium equation for transient heat conduction in grain Ω_1 and Ω_2 can be written as

$$\nabla q(\mathbf{x},t) + \rho_i c_i \frac{\partial \theta(\mathbf{x},t)}{\partial t} = 0 \ \text{ in } \Omega_i, \ i = 1,2 \tag{1}$$

in which $\mathbf{x}$ denotes the position vector; c_i and ρ_i are the heat capacity and mass density of grain Ω_i, respectively. $q(\mathbf{x}, t)$ represents the heat flux, which can be determined by the Fourier's law:

$$q(\mathbf{x},t) = -k_i \nabla\theta(\mathbf{x},t) \ \text{ in } \Omega_i, \ i = 1,2 \tag{2}$$

where k_i is the heat conductivity of grain Ω_i. The associated boundary conditions and initial condition of this problem can be written as

$$\theta(\mathbf{x},t) = \bar{\theta}(\mathbf{x},t) \ \text{ on } \ \Gamma_1 \tag{3}$$

$$q(\mathbf{x},t)\cdot\mathbf{n} = \bar{q}(\mathbf{x},t) \ \text{ on } \Gamma_2 \tag{4}$$

$$\theta(\mathbf{x},0) = \theta_0 \tag{5}$$

in which, $\bar{\theta}(\mathbf{x},t)$ is the specified temperature on boundary Γ_1; $\bar{q}(\mathbf{x},t)$ is the specified heat flux on boundary Γ_2; θ_0 is the initial temperature of the problem domain; $\mathbf{n}$ is the corresponding unit normal vector of the boundary. Besides, the solution of the temperature field should also satisfy the interfacial heat interactive conditions

$$q(\mathbf{x},t)\cdot\mathbf{n}^+ = q_{cz}^+ \ \text{ on } \Gamma_{cz}^+ \tag{6}$$

$$q(\mathbf{x},t)\cdot\mathbf{n}^- = q_{cz}^- \ \text{ on } \Gamma_{cz}^- \tag{7}$$

where $\mathbf{n}^+$ and $\mathbf{n}^-$ are the corresponding outside unit normal vector of Γ_{cz}^+ and Γ_{cz}^- respectively; q_{cz}^+ and q_{cz}^- are the corresponding heat flux across boundary Γ_{cz}^+ and Γ_{cz}^-, respectively.

3. Transient heat conduction analysis by NMM

3.1. NMM approximation

The most appreciative characteristic of the NMM is the dual cover systems, i.e. the MC and the PC. Considering a problem domain with physical identities, such as outside boundaries, joints, holes, material interfaces, etc. The MC is a set of arbitrary shaped small patches that is independent of the physical identities and covers the whole problem domain. After constructing the MC, the MC is further divided by the physical identities into smaller patches. The subdivisions of the MCs are termed as PCs. Different PCs may partially overlap each other and the common areas of the PCs are the manifold elements (ME). The MEs govern the integral areas of the NMM.

For easier understanding, the following simple example is presented. We consider the physical domain in Fig. 2a, which is divided into two parts by the centre red interface. Firstly, two MCs, termed as M_1 and M_2 respectively, are generated to cover the whole problem domain (Fig. 2b). Then, M_1 is further divided into two PCs by the physical identities, i.e. P_1 and P_2; M_2 is disintegrated into two PCs termed as P_3 and P_4, as shown in Fig. 2c. Some of these four PCs are partially overlapping and the whole problem domain is divided into six MEs as shown in Fig. 2d, in which the PCs in the parentheses are the corresponding PCs of each ME.

After generating the cover systems, the local approximation function for each PC can be easily obtained. Originally, the NMM adopts polynomial functions to approximate local displacements of each PC. Similarly, we can also adopt the polynomial functions to approximate the local temperature field of each PC:

$$\theta_i^{pc}(\mathbf{x},t) = \mathbf{P}^T\boldsymbol{\theta}, \ i \in \left[1, n^{pc}\right] \tag{8}$$

where $\theta_i^{pc}(\mathbf{x},t)$ represents the local temperature field in each PC; i is serial number of the PC; n^{pc} denotes the total number of the PC in the