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On singular ES-FEM for fracture analysis of solids with singular stress fields of arbitrary order

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ABSTRACT

The singular edge-based smoothed finite element method (sES-FEM) using triangular (T3) mesh with a special layer of five-noded singular elements (sT5) connected to the singular point, was proposed to model fracture problems in solids. This paper aims to extend the previous studies on singular fields of any order from −0.5 to 0, by developing an analytical means for integration to obtain the smoothed strains. We provide a more efficient practical formulae to estimate the stress intensity factor(SIF) for singular fields of mentioned order. The sT5 element has an additional node at each of the two edges connected to the crack tip, and the displacements are enriched with necessary terms to simulate the singularity. A weakened weak (W2) formulation is used to avoid the differentiation to the assumed displacement functions. The stiffness matrix is computed by using the smoothed strains calculated analytically from the enriched shape functions. Furthermore, our analytical integration techniques reduces the dependency on the order of numerical integration during the computation of the smoothed strain matrix. Several examples have been presented to demonstrate the reliability of the proposed method, excellent agreement between numerical results and reference observations shows that sES-FEM is an efficient numerical tool for predicting the SIF for singular fields.

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1. Introduction

In the seminal publication of the Smoothed Finite Element Method (S-FEM) [1], Liu introduced the strain smoothing technique [2] into the finite element method (FEM) formulation [3,59,85]. It was motivated by the idea of combining the well-established FEM for efficiency and simplicity, with techniques used in the meshfree methods [4] for "softening effects" to improve the accuracy of solutions. The strain smoothing technique was used to stabilize the solutions of the nodal integrated meshfree methods [6]. It was also applied to the natural elements method [7], among other applications [4,5]. The essential idea of the S-FEM is the reconstruction of the strain field using the strain smoothing technique. In the standard Galerkin weak form used in the FEM, the assumed displacement field makes the numerical model "stiff" [4]. In the smoothed Galerkin weak form (which is a typically weakened weak-form or W2 [4]), a smoothed strain is used that brings in softening effects and make the model "softer", while the assumed displacement makes the model stiff. Theoretical study of the S-FEM has proven that the strain smoothed stiffness matrix is always softer than that created in the standard FEM using the same set of nodes, in terms of strain energy norm [7–11]. One thus has an important "knob" to tune the numerical model as desired. This feature of S-FEM stands out from the standard FEM and possesses a number of important features [18], because it allows analysts to "design" models base on their needs. One can now create models for upper bound solutions (for force driven problems) [13], and even close-to-exact solutions in a norm [19]. Due to the softening effects, S-FEM has the capability to perform excellently for highly distorted meshes as well as n-sided general polygonal elements [18]. It does not demand high-quality mesh, as it usually does in the FEM.

The S-FEM is a combination of treatments in the FEM and Meshfree setting [14–18]. The displacement interpolation in S-FEM is based on a standard FEM mesh, and the weak form is evaluated based on the smoothing domains created according to the requirements of the analyst. The art of the S-FEMs lies now on the novel creation of various types of smoothing domains to establish a model of specially desired properties. The smoothing domain can alter the strain field within the elements; it can also bring information from the neighboring elements. Smoothing domains used in S-FEM are the subdomains over which the smoothed strain fields are obtained. They can be created based on cells, nodes, edges or faces of a background FEM mesh, and hence there is a family of S-FEM models. When the smoothing domains are created based on cells, one has the cell-based smoothed FEM (CS-FEM), and when based on nodes, we have NS-FEM. Similarly, we have edge-based smoothed FEM (ES-FEM), face-based smoothed FEM (FS-FEM). There are also combined S-FEM models, like the Alpha-FEM [19]. The ES-FEM

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was found most computationally efficient and having "close-to-exact" stiffness [15]. S-FEM has been widely applied to a number of applications in the domain of continuum mechanics like dynamics [15], plasticity [39–41], plates and shells [20–25], piezoelectricity [26], limit analysis [42], fluid-structure interaction [43], acoustics [37–38] and fracture mechanics [27–36].

Due to the importance in engineering applications, intensive studies have been performed in the field of fracture mechanics over the past decades. The capability to predict fracture in a solid structure is of utmost importance to an analyst, several techniques have been used in the past for computing important parameters, such as the stress intensity factor (SIF) at the tip of cracks and using it in a failure criterion based on fracture toughness of the material. Alongside experiments, the focus has been on the numerical methods to solve fracture problems producing singularity at the crack tip to accurately and efficiently compute the SIF. When a crack is present in a solid, the problem domain becomes non-Lipschitzian domain, and the stress will be singular at the crack tip, resulting in loss of convergence in the numerical solution. Hence special treatments are needed to bring back the convergence rate of the numerical model [4]. A typical technique is to "enrich" the displacement approximation aiming to produce the singular stress field. The common technique is the use of the collapsed quadrilateral element in a FEM mesh, using six-node triangles (T6) or eight-node quadrilaterals (Q8) [3]. Several other known methods are the partition of unity finite element method PUFEM based on the works of Oden et al [44–48], the extended finite element method (X-FEM) [49–55]. More recent works include the efficient remeshing techniques [91–93], the screened-Poisson equation [94,95] and the cracking particles method [96,97]. In the S-FEM, five-noded singular elements have been developed. It can be used in various S-FEM models for simulating the singular stress field near the crack tip, including the ES-FEM model using three-noded linear triangles elements (T3). It is known as the s-ES-FEM-T3 [32] and has been proven with good performance for fracture mechanics problems. It uses a base mesh of linear triangular elements, unlike the standard singular FEM that needs a mesh of quadrilateral elements (T6 or Q8).

Many solid structures are built with sharp corners rather than perfect cracks, and hence the problem domain is also a typical non-Lipschitzian [4]. In such cases, the order of singularity of the stress field is different from the idealized cracks. Sharp re-entrant corners can still cause stress singularities and thus reduces the longevity of the structure. Thus, a careful study of the stress singularity field around shape corners is of significant importance. However, very little work has been done in the non-Lipschitzian domain, compared to the idealized sharp crack configuration. Since the pioneer paper of Williams [60], which identifies the stress intensification at the vertex of sharp re-entrant corners, a handful number of studies have been performed to determine the order of singularity at a crack tip and subsequently compute the stress intensity factor. Sinclair [61–63] has studied the singularity in linear elastic fracture mechanics and has proposed numerical techniques to obtain stress singularity exponent of singular geometries. On the computation of stress intensity factors in singular re-entrant corners, the earliest research was for wood structures with different size specimens [64–66]. Sinclair and Kondo pursued a stress concentration approach to discuss a generalized stress intensity factor at re-entrant corners [67]. Sinclair et al. [68] also developed an approach to evaluate a contour integral extending the work of Stern et al. [69]. Independently, Carpenter [70] used the reciprocal work contour integral method of Stern [69] to compute stress intensity factors at corners. Carpenter has subsequently studied further in these domains and suggested other techniques for determination of fracture mechanics parameters [71–73]. Sinclair [74] have discussed and compared both these approaches in details. Further work has been done by Neville [75] who proposed a statistical approach to failure prediction, Gross, and Mendelson [76], Knesl [77], Carpentri [78] who performed three-point bending of beams to experimentally co-relate failure to the re-entrant corner angle has performed a detailed study of intensity at sharp corner.

Lin and Tong [79], Dunn [80] and Seweryn [81] have done numerical studies in fracture mechanics of notches, using FEM. Special elements have been developed to produce the singularity at the notches. The existing standard approach to solving a re-entrant corner includes,

- A very dense mesh at the re-entrant corner tip (Dunn [80]).
- Degenerated asymptotic finite elements (Tracy [82], Pu et al. [83]).
- Hybrid finite elements (Lin and Tong [79]).
- Analytical finite elements (Givoli [84]).

In the so-called s-ES-FEM method, the singularity at the corner can be easily reproduced by directly adding in a proper singularity term in the basis function for the interpolation of the displacements. This is because the S-FEM uses W2 formulation and hence the simple point interpolation methods can be used, and the resultant interpolants are not subjected to differentiation and not mapping is required. The energy based interaction integral method can also be conveniently used in an S-FEM model to calculate the SIFs [31]. The concept of s-ES-FEM is straightforward, theoretically rigor, easy for implementation, and hence it is ideal for study the fracture mechanics problems. As mentioned earlier, previous studies have been performed for geometries with L-Shaped notches [27], in our work, we extend these to the more exhaustive study of the domain of linear elastic singular mechanics for arbitrarily shaped notches.

In this paper, we intend to propose a few extensions of s-ES-FEM for linear elastic fracture mechanics of sharp cracks. We show that in the singular elements around the crack tip, there is no need to use more than one smoothing domain [32], if a certain number of Gauss points are used to compute the smoothed strains. Further, we propose a method based on analytical integration along the edges of the smoothing domains, thereby eliminating the requirement of Gauss points in general. We also study the performance of s-ES-FEM in geometries with a singularity of order −0.5 to 0 with a base mesh of T3 elements together with and a layer of T5 elements around the notch-tip. The stability, accuracy, and efficiency of this technique are examined through a number of examples. We have also confirmed that the SIF values calculated are stable, path independent and highly accurate.

The paper is outlined as follows. In Section 2, general ES-FEM and two-dimensional singular problems are described. In Section 3, a brief idea of singular ES-FEM and numerical techniques to determine the stress singularity orders are given. The solution procedure is summarized in Section 4. Numerical examples are provided in Section 5.

2. Overview of linear fracture mechanics and ES-FEM

2.1. Governing equations

Consider a two-dimensional (2D), homogeneous isotropic linear elastic solid defined in domain Ω bounded by $\mathbf{\Gamma}$ ($\mathbf{\Gamma} = \mathbf{\Gamma}_U \cup \Gamma_t$ and $\mathbf{\Gamma}_U \cap \Gamma_t = 0$) containing a traction free re-entrant corner Γ_c as shown in Fig. 1. The equilibrium equation is given as follows [86]:

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f}^b = 0 \; in \; \Omega \tag{1}$$

where ∇ is the divergence operator, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, and $\boldsymbol{f}^b$ is the body force.

The Dirichlet and Neumann boundary conditions are given as:

$$\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}, t) \; on \; \Gamma_U \tag{2}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \mathbf{f}^t \; on \; \Gamma_t \tag{3}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = 0 \; on \; \Gamma_t \tag{4}$$

where $\boldsymbol{n}$ being the outward normal vector on the boundary Γ and $\bar{\mathbf{u}}$ is the prescribed displacement on the displacement boundary Γ_U

The stress–strain relation is given by the constitutive equation

$$\boldsymbol{\sigma} = \mathbf{D}\epsilon \tag{5}$$