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JID: EABE [m5GeSdc;November 8, 2017;4:7]

Engineering Analysis with Boundary Elements 000 (2017) 1–7

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Expanding element interpolation method for analysis of thin-walled structures

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ARTICLE INFO

Keywords:
Thin-walled structures
Nearly singular integrals
Expanding element interpolation method
Boundary element method

ABSTRACT

An expanding element is obtained by adding virtual nodes along the perimeter of the traditional discontinuous element. There are two kinds of shape functions in the expanding element: (i) the raw shape function, i.e. shape function of the original discontinuous element, involving only inner nodes; (ii) the fine shape function, which involves all the nodes including inner nodes and the newly added virtual nodes. The polynomial order of fine shape functions of the expanding elements increases by two compared with their corresponding raw shape functions. In this paper, we apply the expanding element interpolation method to analysis of thin-walled structures. An adaptive element subdivision method for evaluating nearly singular integrals is proposed. Numerical results have demonstrated that our method has high level of accuracy and is able to analyze very slender structures with the aspect ratio up to 1e6.

1. Introduction

Thin-walled structures, such as various thin films in electronic devices, sensors and actuators in smart materials, and coatings on machine components, widely appear in engineering application. Accurate and efficient numerical analysis of these structures has been a challenging task. The finite element method (FEM) is a successful tool to analyze the thin-walled structures using plate and shell elements in most applications. But the plate and shell elements are based on plate and shell theories in which many assumptions about the geometry, loading and deformation of structure are introduced. While using brick elements in the FEM, large number of elements are required due to the aspect ratio limitations of the elements.

The boundary element method (BEM) [1–8] is a more suitable method for numerical analysis of thin-walled structures. This is because, in BEM analysis, only the surface of a body needs to be discretized and accurate results for stress can be obtained without shell assumption. This is particularly beneficial when dealing with connections between thin-walled parts and bulky blocks within a complicated structure. In addition, the trial functions in the FEM formulation must be at least C^0-continuous which is not required in the BEM. This feature is significantly important for the BEM to be superior to the FEM. However, how to make full use of this feature has been a long-standing issue in the BEM community [9], because the continuous and discontinuous elements each have their own advantages and disadvantages.

When using the discontinuous elements, many advantages are provided, for example, simplifying the assembly of the system equations, the mesh generation and the computation of the 'free' terms appearing in the integral equations. But for the same level of accuracy, the number of degrees of freedom is larger, thus more CPU time and memory capacity are required. For the continuous elements, the C° continuity can be guaranteed, but not the C^1 continuity which is necessary for hypersingular integral equation [10,11]. In addition, the corner problems [12] must be considered when using the continuous elements.

To unify the continuous and discontinuous elements, a new expanding element is presented. The expanding element is achieved by adding virtual nodes along the perimeter of the traditional discontinuous element. The inner nodes of discontinuous element are called as source nodes. The boundary integral equation is collocated at the source node, only. There are two kinds of shape functions in the expanding element: the raw shape function and the fine shape function. The raw shape functions are used to build relationship between the virtual nodes and source nodes. While the fine shape functions are used for interpolating boundary field variables. With the expanding element, both continuous and discontinuous fields on the domain boundary can be accurately approximated, and the interpolation accuracy increases by two orders compared with the original discontinuous element.

In this paper, we apply the expanding element interpolation method to analyze thin-walled structures. Accurate and efficient evaluation of nearly singular integrals is of crucial importance for solving thin structures problems. Various methods have been proposed to cope with

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https://doi.org/10.1016/j.enganabound.2017.10.014
Received 3 April 2017; Received in revised form 29 August 2017; Accepted 26 October 2017
Available online xxx
0955-7997/

Please cite this article as: J. Zhang et al., Engineering Analysis with Boundary Elements (2017), https://doi.org/10.1016/j.enganabound.2017.10.014

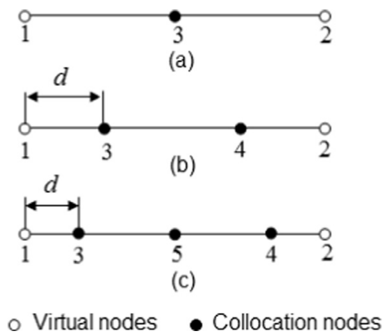

Fig. 1. (a) Expanding constant element; (b) expanding linear element; (c) expanding quadratic element.

nearly singular integrals, such as element subdivision method [13,14], analytical and semi-analytical method [15], exponential transformation [16,17], distance transformation [18,19] and sinh transformation [20–22]. Among these methods, the element subdivision method is a more stable and universal method. An adaptive element subdivision method for evaluating nearly singular integrals is proposed in this paper. In this method, the integration element is divided into two equal sub-elements according to the location of the source node, and it is performed in local coordinate system of the element. With the proposed method, the nearly singular integrals can be accurately evaluated even when the source node is very close to the integration element. Furthermore, this method is independent of the problem to be solved.

This paper is organized as follows. Section 2 presents the expanding element interpolation method. In Section 3, the assembly of the system of linear algebraic equations and the nearly singular integration scheme are described. Numerical examples are given in Section 4. The paper ends with conclusions in Section 5.

2. The expanding element interpolation method

The expanding element interpolation method is introduced in detail in this section.

2.1. The expanding elements

The expanding element is obtained by collocating virtual nodes along the perimeter of the traditional discontinuous element as shown in Fig. 1. There are two kinds of shape functions in the expanding element: raw shape function and fine shape function. The raw shape function is the shape function of the original discontinuous element. The fine shape function is constructed by the virtual nodes and the inner nodes of the discontinuous element. The raw shape functions and fine shape functions of the expanding constant, linear and quadratic element are as follows:

$$\begin{cases} N_3^r = 1 \\ N_1^f = -0.5\xi(1-\xi) \\ N_2^f = 0.5\xi(1+\xi) \\ N_3^f = (1-\xi)(1+\xi) \end{cases} \tag{1}$$

$$\begin{cases} N_3^r = -\dfrac{[\xi-(1-d)]}{2(1-d)} \\ N_4^r = \dfrac{[\xi+(1-d)]}{2(1-d)} \\ N_1^f = -\dfrac{[\xi+(1-d)][\xi-(1-d)](\xi-1)}{2d(2-d)} \\ N_2^f = \dfrac{[\xi+(1-d)][\xi-(1-d)](\xi+1)}{2d(2-d)} \\ N_3^f = \dfrac{[\xi-(1-d)](\xi+1)(\xi-1)}{2d(1-d)(2-d)} \\ N_4^f = -\dfrac{[\xi+(1-d)](\xi+1)(\xi-1)}{2d(1-d)(2-d)} \end{cases} \tag{2}$$

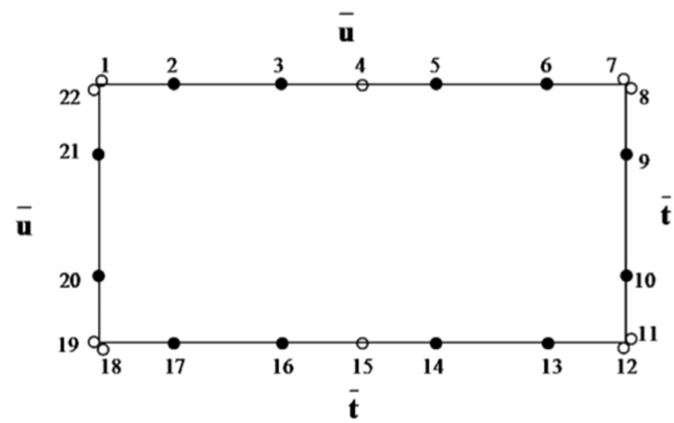

Fig. 2. Example of new interpolation method by the expanding linear elements.

$$\begin{cases} N_3^r = \dfrac{\xi[\xi-(1-d)]}{2(1-d)^2} \\ N_4^r = \dfrac{\xi[\xi+(1-d)]}{2(1-d)^2} \\ N_5^r = -\dfrac{[\xi+(1-d)][\xi-(1-d)]}{(1-d)^2} \\ N_1^f = \dfrac{[\xi+(1-d)][\xi-(1-d)](\xi-1)\xi}{2d(2-d)} \\ N_2^f = \dfrac{[\xi+(1-d)][\xi-(1-d)](\xi+1)\xi}{2d(2-d)} \\ N_3^f = -\dfrac{[\xi-(1-d)](\xi+1)(\xi-1)\xi}{2d(2-d)(1-d)^2} \\ N_4^f = -\dfrac{[\xi+(1-d)](\xi+1)(\xi-1)\xi}{2d(2-d)(1-d)^2} \\ N_5^f = \dfrac{[\xi+(1-d)][\xi-(1-d)](\xi+1)(\xi-1)}{(1-d)^2} \end{cases} \tag{3}$$

where N with the superscripts r and f refer to the raw and fine shape functions of the expanding elements, respectively.

The fine shape functions are used for interpolating boundary field variables. From Eqs. (1)–(3), it can be seen that the interpolation accuracy increases by two orders compared with the original discontinuous element. The relationships between the virtual nodes and source nodes are built up by the raw shape functions.

2.2. New interpolation method by the expanding element

Because virtual nodes are not used as source nodes, how to get the nodal values of virtual nodes is very crucial for the implementation of the new method. In the following example, the calculation of values at virtual nodes is described in detail.

Fig. 2 shows a rectangular domain discretized by 6 expanding linear elements with 12 source nodes and 10 virtual nodes. These elements are used to interpolate displacements and tractions on the boundary. $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}$ in Fig. 2 represent the known displacements and tractions, respectively.

For interpolating known boundary variables with the expanding elements, the nodal values of virtual nodes equal to the corresponding boundary conditions. For instance, $\mathbf{u}_4 = \bar{\mathbf{u}}_4$ in Fig. 2. Thus, more accurate boundary conditions can be imposed. This is particularly beneficial when dealing with thin-walled structures with short edges.

When interpolating unknown boundary variables, the nodal values of virtual nodes equal to the average of extrapolation values by the raw shape functions of their connecting elements. Taking $\mathbf{t}_4$ in Fig. 2 for example,

$$\mathbf{t}_4 = \frac{1}{2}\left[N_2^r(1)\mathbf{t}_2 + N_3^r(1)\mathbf{t}_3 + N_5^r(-1)\mathbf{t}_5 + N_6^r(-1)\mathbf{t}_6\right] \tag{4}$$