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Effect of shear deformation on the buckling parameter of perforated and non-perforated plates studied using the boundary element method

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ABSTRACT

The buckling parameter is a non-dimensional value that typically represents the type of plate boundary condition (e.g., clamped edge, free edge) and is obtained from the critical load and geometrical data. This study investigates the variations in the buckling parameter based on plate slenderness considering the effect of shear deformation in the bending model used for buckling analyses. An alternative boundary element formulation using two integrals containing the geometrical non-linearity (GNL) effect, with one computed on the domain and the other computed on the boundary, is employed. The kernels of integrals related to the GNL effect contain the first derivatives of deflection instead of the second derivatives, and no relation is required for the derivatives of the in-plane forces. This formulation improves the numerical model for free edges or symmetry conditions corresponding to the relationship of one of the natural conditions of the buckling problem to the boundary integral containing the GNL effect. The values obtained for the buckling parameters in plates containing or not containing a hole are compared with the expected values from the literature.

1. Introduction

The present boundary element formulation for plate-buckling analyses considers the shear deformation effect to improve the bending model accuracy, as shown by Reissner for the assessment of stress concentration around holes [1] and by Mindlin for wave propagation analyses considering short wavelengths [2]. The geometric non-linearity (GNL) effect is considered according to the development presented by Timoshenko and Woinowsky-Krieger [3]. Although classical plate theory was used in [3], several studies in the literature have adopted this formulation in buckling analyses for thin or moderately thick plates considering the effect of shear deformation.

Jones presented a literature review up to 1989 on the buckling of thin rectangular plates in [4]. Dawe and Roufaeil [5] discussed the GNL effect in the bending of plates while considering the effect of shear deformation based on the first study presented by Hermann and Armenakas [6], and they stressed the importance of introducing the derivatives of rotations beyond the derivatives of deflections in the potential energy density associated with in-plane forces. Mizusawa [7] showed that the effect of the derivatives of rotations is greater for certain types of boundary conditions, whereas they are not significant in others for which the derivatives of deflection were sufficient. Smith [8] proposed a finite element formulation including both derivatives on displacements in the buckling of thick plates. Doong [9] and Matsunaga [10] studied improvements to the model representing the effect of shear deformation in plate bending, where the buckling analyses considered the derivatives of rotations and deflections combined with a high-order theory representing the effect of shear deformation. Nevertheless, several studies, e.g., Shufrin and Eisenberg [11] and Kitipornchai and Xiang [12], employed only derivatives of the deflection including or not including the high-order theory to consider the effect of shear deformation. Srinivast and Rao [13] employed three-dimensional elasticity theory to perform vibration and buckling analyses of thick orthotropic plates and laminates.

Levy et al. [14] studied the instability of perforated plates with a central hole. Schlack and Alois [15] computed the critical edge displacement of a simply supported pierced plate under uniform edge displacement. Yang [16] showed that the critical plate load is reduced when a square hole is considered instead of a circular hole. Brown and Yettram [17] illustrated the changes in the value of the buckling parameter for different load combinations according to the ratio between the diameter of the hole and the plate side. Shakerley and Brown [18] studied plate buckling with eccentrically positioned holes. El-Sawy and Nazmy [19] used the finite element method to assess the buckling parameter value for uniaxial loaded plates according to the ratio between the diameter of the hole and the plate side. Komur and Sonmez [20] studied the buckling of perforated plates with a circular hole under linearly varying in-plane load. Maiorana et al. [21] presented a buckling analysis of perforated plates under concentrated in-plane loads. Nejad and Shanmugam [22] presented a buckling analysis of uniaxial loaded perforated skew plates. Jayashankarbabu and Karisiddappa [23] presented the buckling of perforated plates considering the shear deformation effect instead of the classical plate theory used in studies [14–22].

Several boundary element formulations presented in the literature performed buckling analyses with the classical theory or Reissner-Mindlin models. Bezine et al. [24] presented plate-buckling analyses

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using the classical theory with the domain integral introducing the GNL effect. Liu [25] used the classical theory for the buckling analysis of circular plates and employed deflections instead of the derivatives of deflections in the domain integral related to the GNL effect. Manolis et al. [26] presented a fundamental solution that included the effect of the uniform in-plane force on the domain to eliminate the domain integral related to the GNL effect in buckling analyses with the classical theory. Tanaka and Miyazaky [27] presented a boundary element formulation to perform buckling analyses of assembled plates using the classical theory and employed domain integrals for the GNL effect. Elzein and Syngellakis [28] employed the dual reciprocity method (DRM) for the GNL effect in buckling analyses with the classical theory. Nerantzaki and Katsikadelis [29] studied plate buckling considering variable plate thickness according to the classical theory with a BEM-based method named the analog equation method. Lei et al. [30] formulated an integral equation for the geometrically nonlinear behavior of Reissner plates, where the domain integral for the GNL effect was discretized into constant triangular cells. Marczak [31] employed constant rectangular cells in the domain integral discretization for the GNL effect for buckling analyses using the Reissner model. The buckling analyses using the Reissner model in Purbolaksono and Aliabadi [32] presented a comparison of the results obtained with the DRM versus domain integrations to account for the GNL effect. Doval et al. [33] employed the radial integration method (RIM) for integrals related to the GNL effect in the analysis of composite laminate plates under non-uniform stress fields using the classical theory. Chang-Jun and Rong [34] performed buckling and post-buckling analyses of perforated plates with the BEM using the classical plate theory.

The alternative boundary element formulation in this study employs two integrals containing the GNL effect, with one computed on the domain and the other computed on the boundary. The first derivatives of deflection were used in kernels of integrals related to the GNL effect instead of the second derivatives; no relation is required for the derivatives of in-plane forces. This formulation improves the numerical modeling to treat the free edges or symmetry conditions corresponding to the relationship between one of the natural conditions of the buckling problem to the boundary integral containing the GNL effect. The critical loads were obtained with this formulation using an elastodynamic fundamental solution derived from [37] in problems with in-plane forces assumed to be invariant with time and deflection derivatives dependent of the harmonic solution [35,36]. The changes in value of the buckling parameter according to the plate thickness of non-perforated plates were presented in [38] for certain types of boundary conditions with the static fundamental solution [39]. The numerical implementation employed quadratic shape functions to approximate displacements, plate-rotations, distributed shears and moments in the boundary elements; however, constant elements were used to discretize both integrals related to the GNL effect. Constant elements were the lower type of element used to evaluate the behavior of this formulation. An algebraic manipulation using both integrals with the GNL effect corresponds to performing integrations only on the sides of cells inside the domain in problems with no free edges. The inverse iteration and Rayleigh quotient were used to compute the lowest eigenvalue with the corresponding eigenvector. The values obtained for the buckling parameters in plates containing or not containing a hole are compared with the expected values from the literature.

2. Natural conditions and boundary integral equations

The constitutive equations for an isotropic and homogeneous plate material are

$$M_{\alpha\beta} = D\frac{(1-\nu)}{2}\left(\psi_{\alpha,\beta} + \psi_{\beta,\alpha} + \frac{2\nu}{1-\nu}\psi_{\gamma,\gamma}\delta_{\alpha\beta}\right) + \delta_{\alpha\beta}qRE \tag{1}$$

$$Q_\alpha = D\frac{(1-\nu)}{2}\lambda^2\left(\psi_\alpha + w_{,\alpha}\right) \tag{2}$$

with

$$\lambda^2 = 12\frac{\kappa^2}{h^2};\ RE = \frac{\nu}{\lambda^2(1-\nu)}$$

The plate has a uniform thickness *h, D* is the flexural rigidity, ν is Poisson's ratio, w is the deflection, ψ_α is the plate rotation in direction α and $\delta_{\alpha\beta}$ is the Kronecker delta. The product qRE in Eq. (1) corresponds to the linearly weighted average effect of the normal stress component in the thickness direction and should be considered in Reissner's model [1] but not in Mindlin's model [2], in which it should be considered null. The shear parameter κ^2 is equal to 5/6 and $\pi^2/12$ for the Reissner and Mindlin models, respectively. Eqs. (1) and (2) employ a unified notation for the Reissner and Mindlin models and are written in the convention adopted in this study, i.e., Latin indices take on values {1, 2 and 3} and Greek indices take on values {1, 2}.

The natural conditions and equilibrium equations can be obtained using the calculus of variations [40,41]. The energy functional of the plate under static loads is given by

$$\begin{aligned}\Pi = &\int_\Omega \left\{\frac{D(1-\nu)}{4}\left[{\psi_{\alpha,\beta}}^2 + \psi_{\alpha,\beta}\psi_{\beta,\alpha} + \frac{2\nu}{(1-\nu)}{\psi_{\gamma,\gamma}}^2 + \lambda^2\left(\psi_\alpha + w_{,\alpha}\right)^2\right]\right\} d\Omega \\ &+ \cdots - \int_\Omega qw d\Omega - \int_{\Gamma_{np}} \left(EQw + EM_\alpha\psi_\alpha\right) d\Gamma \\ &- \int_\Omega \frac{1}{2}\left(N_{\alpha\beta}w_{,\alpha}w_{,\beta}\right) d\Omega\end{aligned} \tag{3}$$

Eq. (3) represents the energy functional of the plate in the complete form, which is similar to that presented in [12]. The first integral (domain integral) is the strain energy, and the last integral is the potential energy due to in-plane compression forces. The second and third integrals are the potential energy due to out-of-plane loads: q is the normal load distributed on the domain, and EM_1, EM_2 and EQ are the couple in direction 1, the couple in direction 2 and the normal load, respectively, which are distributed on the free edge (Γ_{np}). Displacements *w*, ψ_1, and ψ_2 are not prescribed on the free edge (Γ_{np},). The general function from Eq. (3) to be minimized with the calculus of variations is

$$\Pi = \int_\Omega F\left(w, \psi_1, \psi_2, w_{,1}, \psi_{1,1}, \psi_{2,1}, w_{,2}, \psi_{1,2}, \psi_{2,2}\right) d\Omega \tag{4}$$

The minimization of Eq. (4) leads to the following equations, i.e., Euler equations [40,41]:

$$\frac{\partial F}{\partial \psi_\alpha} - \frac{\partial}{\partial x_\beta}\left(\frac{\partial F}{\partial \psi_{\alpha,\beta}}\right) = 0 \tag{5}$$

$$\frac{\partial F}{\partial w} - \frac{\partial}{\partial x_\beta}\left(\frac{\partial F}{\partial w_{,\beta}}\right) = 0 \tag{6}$$

The equilibrium equations obtained from Eqs. (5) and (6) after using the constitutive equations are

$$M_{\alpha\beta,\beta} - Q_\alpha = 0 \tag{7}$$

$$Q_{\alpha,\alpha} + q + \frac{\partial}{\partial x_\alpha}\left(N_{\alpha\beta}\frac{\partial w}{\partial x_\beta}\right) = 0 \tag{8}$$

The natural conditions introduce requirements on the free edge (Γ_{np}), which is the boundary portion without the prescribed displacements. The variations on displacements are not null on Γ_{np} (i.e., $\delta w \neq 0$ and $\delta\psi_\alpha \neq 0$) and yields

$$\left(\frac{\partial F}{\partial w_{,\alpha}}n_\alpha\right)\delta w = 0 \overset{yields}{\rightarrow} \left(\frac{\partial F}{\partial w_{,\alpha}}n_\alpha\right) = 0 \tag{9}$$

$$t_3 = EQ - n_\alpha N_{\alpha\beta}w_{,\beta} \tag{10}$$

$$\left(\frac{\partial F}{\partial \psi_{\alpha,\beta}}n_\beta\right)\delta\psi_\alpha = 0 \overset{yields}{\rightarrow} \left(\frac{\partial F}{\partial \psi_{\alpha,\beta}}n_\beta\right) = 0 \tag{11}$$