Engineering Analysis with Boundary Elements 85 (2017) 116–126

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Boundary Element Method applied to decision-making problems involving geometric variabilities in topology optimization

Hugo Luiz Oliveira[a,*], Alaa Chateauneuf[b,c], Edson Denner Leonel[a]

[a] University of São Paulo, São Carlos School of Engineering, Department of Structural Engineering, Avenida Trabalhador SãoCarlense, 400 Centro, São Carlos, SP 13566-590, Brazil
[b] Université Clermont Auvergne, Institut Pascal, BP 10448, F-63000 Clermont-Ferrand, France
[c] CNRS, UMR 6602, IP, F-63178 Aubière, France

ARTICLE INFO

Keywords:
Boundary Element Method
Geometric variability
Surface Response Method
Decision-making process

ABSTRACT

Topology Optimization is a useful approach in the early stages of engineering design because it provides structural layout satisfying the performance requirements with the less material amount. Optimal topology solutions may not be unique for the same initial conditions. Local minima may be achieved depending on the design parameters. In such cases, deterministic criteria are often used for choosing the most convenient solution. In the present study, a probabilistic framework for obtaining additional information from the design is proposed, in order to improve the decision-making process. This methodology takes into consideration the less sensitive topology with respect to the geometric variabilities. The framework is based on a novel coupling of Boundary Element Method, Surface Response Method and Probability theory. Two engineering applications are utilized for demonstrating the proposed methodology. The results illustrate that the topology chosen based on the straightforward deterministic criterion may not be sufficient to determine a robust solution from the engineering point of view.

1. Introduction

Topology Optimization (TO) has been recognized as a valuable tool for engineers during the design phase. This approach enables the determination of the structural layout based on design constraints. It is well known that TO problems are non-convex and the determination of global minimum is a complex task. Usual gradient based approaches are inherently susceptible to local minima, which are frequently not unique. In these cases, the analyst is designated to make choices among multiple feasible solutions that satisfy the required constraints. If all of them have the same volume material, for instance, which one would be the best topology? To make this choice, it is important to have a criterion in mind. An immediate criterion would be the compliance value. Thus, the best solution is the one that leads to the least compliance value. This is an example of deterministic criterion. The present study investigates the efficiency of the deterministic criteria based on the compliance value. Would it lead to suitable solutions from the engineering point of view?

To answer this question, a framework will be proposed in this study, which is based on the coupling of Boundary Element Method (BEM), the Surface Response Method (SRM) and the Probability theory. This methodology is stated from the shape sensitivity analysis which utilizes only boundary information. Thus, the general framework is suitable for BEM applications and it can be applied to extract additional information from the topological design.

The present study represents the beginning steps on the direction of bringing BEM closer to the emerging field of Probability applied for engineering, exploring its inherit capabilities.

2. Literature review

The search for improved structural layouts is a challenging and active research field. This problem can be formulated in terms of moving boundaries [1] where the Level Set Method (LSM) and the BEM find their advantages. The LSM provides a convenient function for the parameterization of the design domain [2,3]. This key feature can be explored in other fields of engineering such as sound scattering [4]. The clear and smooth definition of the boundaries [5] represent an attractive advantaged when compared to hard-kill approaches [6]. Zero order searches [7,8] have also been explored in a BEM framework showing results comparable to traditional gradient-based methodologies. The coupling between LSM and BEM is also beneficial for problems involving design dependent boundary conditions [9]. More recent formulations have focused on increasing the BEM computational performance [10]. All these BEM-based investigations have one common point, which is assuming

* Corresponding author.
E-mail address: hugoitaime@usp.br (H.L. Oliveira).

https://doi.org/10.1016/j.enganabound.2017.09.016
Received 10 July 2017; Received in revised form 27 September 2017; Accepted 27 September 2017

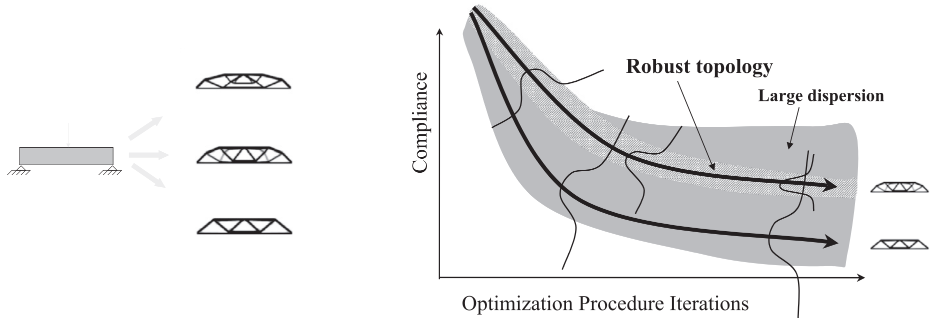

Fig. 1. Topology optimization with different reliability levels [25].

the complete knowledge about the set of input parameters. Thus, they can be classified as deterministic approaches.

Deterministic techniques have been utilized successfully in engineering field for reducing manufactory costs and enhancing structural performances. However, these techniques do not account for the intrinsic random nature of the engineering problems. The minimum compliant typologies lead to higher levels of material solicitation. Thus, these structures are useful when it is possible to maximize its lifespan keeping acceptable safety levels. Concerning TO, two fronts of investigation appears frequently in the literature.

The first line of research is related to TO of structures limited to some probability of failure. It is the so-called Reliability Based Topology Optimization (RBTO). This field has been developed along the last years showing that the probability of failure has major impact on the final shape [11]. Alternative techniques have been developed in order to increase computational efficiency by single loops [12], instead of classical double loop approaches. The interest of the reliability approach in TO is to handle uncertainties as a control criterion for topology selection. In fact, the reliability constraint enables choosing a robust structural topology. Usually, the comparison in deterministic topology optimization is only related to the minimum expected compliance, without observing the solution dispersion. The principle of reliability-based and robust topology consists in defining the topology that is less sensitive to the system uncertainties.

In general, RBTO applies the same objective function utilized on deterministic techniques. The RBTO depends on the probability of failure constraints and how they affect the geometrical sensitivities. The probability of failure can be determined by classical approaches (such as FORM – First Order Reliability Method) which depend on the accurate calculation of very low probabilities. This is not a simple task for general engineering applications. An alternative approach, is the so-called Performance Measure Approach (PMA) [13]. A comparison between PMA and RIA (Reliability Index Approach) shows few discrepancies among the optimal topologies [14].

Bidirectional Evolutionary Topology Optimization (BESO) can be applied for RBTO. In this case, special attention should be addressed to filters, since the *sensitivity number* may not lead to convergent topologies. Eom et al. [15] proposed the use of precedent information for updating the current sensitivity number parameters. Metamodelling has been advised for increasing computational efficiency. This strategy leads to coherent topologies also verified by other investigations [16]. Multi-objective BESO/RBTO has found industrial applications [17] including heterogeneous components [18]. Because BESO analysis involves high computational effort, some works have devoted attention for increasing its efficiency [19]. These studies have showed BESO capabilities for RBTO.

System Reliability Analysis can be utilized for RBTO coupled with second order methods (SORM) [12] showing similar results to those obtained via first order methods (FORM). Zhao et al. [20] compare two different approaches, i.e. PMA and SORA (Sequential Optimization Reliability Assessment). The authors propose the use of Surface Responses for decreasing computational efforts. The results show that SORA can be advantageous because it has faster convergence. The LSM has also been investigated [21]. The preliminary results show that accuracy still needs to be improved. The Segmental Multi-Point Linearization (SML) technique was proposed as alternative to FORM for sensitivities calculation with promising results in terms of convergence [22].

A compilation of existing techniques for RBTO was presented by Zhao et al. [23]. Although most of these techniques are based on the Finite Element Method (FEM), they are extensible to BEM. It is worth remarking that the optimal topologies determined accounting for uncertainties are more reliable than deterministic ones. This conclusion has major impact on aircraft structures [24], and other mechanical industry structures. In addition, it is verified that metamodelling is an important tool for decreasing computational time. The present study will take these aspects into consideration.

The second line of investigation is dedicated to maximizing the non-deterministic system robustness. In this case, the central idea is determining the structural topology less sensitive to the randomness involved. It is the so-called Robust Topology Optimization (RTO). Fig. 1 illustrates the difference between deterministic optimization and robust topology [25]. Both deterministic and robust solutions are obtained for the same initial problem. Although deterministic topologies manifest lesser compliance, the performance dispersion observed is larger in general. Robust solutions manifest lesser mechanical performance dispersion when uncertainties are present. Thus, they are more reliable from an engineering point of view.

Two categories of RTO can be collected from the literature. The first one, and also the most common, is using stochastic methods. In these cases, part of the input parameters is assumed to be random following a predefined probability law. The second manner is a deterministic variation named as worst-case scenario [26,27]. The procedure involves the determination of the structures that can support the set of the most unfavourable conditions (for instance, the worst geometric errors regardless its probability of occurrence). The worst-case scenarios lead to optimal topologies more conservative if compared to stochastic solutions [28].

The LSM was also applied on RTO investigations. Chen et al. [29] propose an LSM based algorithm for handling random field functions and the loading conditions via Karhunen–Loève (K–L) decomposition. The effects of load uncertainties over robust topologies was investigated from several approaches [30–34]. The results show that robust topologies can differ significantly from their correspondent deterministic ones. The LSM was also utilized for handling cases of uncertainties related to shape [35]. The cases involving geometric nonlinearities were presented by Jansen et al. [36] showing that stable robust topologies have less amount of members under compression.

Two aspects are still recognized as challenges for the development of RTO algorithms [37]. At first, the efficient representation of uncertainties concerning structures only on their design phase. The second difficulty arises when the large number of design variables need to be processed with acceptable computational time. It is worth mentioning