Engineering Analysis with Boundary Elements 85 (2017) 158–164

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A parallelizable direct solution of integral equation methods for electromagnetic analysis

Kechen Wang, Mengmeng Li*, Dazhi Ding, Rushan Chen

Department of Communication Engineering, Nanjing University of Science and Technology, Nanjing, China

ARTICLE INFO

Keywords:
Hierarchical fast direct solver
Integral equation
Multiscale problems

ABSTRACT

A parallelizable direct solution of integral equation methods is proposed for electromagnetic scattering analysis in low to intermediate frequency regime. There are mainly two parts of the proposed direct solution: forward decomposition and backward substitution. For the forward decomposition, the dense impedance matrix is decomposed of the product of several block diagonal matrices implicitly, which is shown to be $O(N\log^2 N)$ for both memory and CPU time cost. The final solutions are obtained with several matrix vector products (MVPs) in the part of backward substitution with $O(N\log^2 N)$ complexity as well. Both forward decomposition and backward substitution can be parallelized because of the group independence. Furthermore, an effective preconditioner with a reasonable selection criterion of the diagonal blocks region is proposed to accelerate the convergence of the iterative solver. The proposed solution is independent of the Green's function, and it is suitable for all the integral equation methods. Without loss of generality, the solution is proposed to solve the electric field integral equation (EFIE) in this work. Numerical tests demonstrate the effectiveness of the proposed solution for the electromagnetic analysis, especially for multiscale structures.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

The integral equation methods [1] are popularly applied in the area of electromagnetic analysis. Many solution methods [4–17,26–28] have been proposed to obtain the required solutions of integral equations. These methods are mainly divided into two kinds, the iterative solution method and the direct solution method.

The iterative solution methods such as the generalized minimum residual (GMRES) [2] or the bi-conjugate gradient (BiCG) [3] need one or more MVPs at each iteration. The computational complexity of MVP can be reduced by many fast algorithms, such as multilevel fast multipole algorithm (MLFMA) [4,5], the matrix decomposition algorithm (MDA) [6–8], adaptive cross approximation (ACA) method [9,10] and nested equivalent source approximation (NESA) method [11], etc. However, the computational efficiency of the iterative methods is restricted by the number of iterations which is unpredictable, especially for the multiscale problems. To accelerate the convergence of the iteration solvers, many preconditioning techniques [22–28] have been proposed in recent decades. However, the iterative solution needs to be resumed for each right-hand-side (RHS) vectors for the multiple excitations problems (monostatic RCS problems), which is time-consuming.

In order to overcome the unpredictable problem of the iteration number arising from iterative solver, the direct solution methods have been studied in recent years. The impedance matrix can be inversed by the direct solution method. The solution of the matrix equations can be obtained with a simple backward substitution for given excitation vectors. However, The conventional direct solution method like the Gaussian elimination or LU decomposition cannot be effective for electrical-large electromagnetic problems, which is $O(N^3)$ (N is the number of unknowns) computational complexity. Therefore, many fast direct solution methods [12–17] have been proposed during recent several years. The compressed block decomposition (CBD) [12] and multiscale compressed block decomposition (MSCBD) [13] based on a block-wise subdivision of the impedance matrix of EFIE are proposed for accelerated direct solution of electromagnetic analysis with computation complexity of $O(N^2)$ for computation cost and $O(N^{1.5})$ for memory. The H^2-matrix-based solver [14] is developed to solve the dense system of linear equations for static capacitance extraction problems with linear complexity. In [15], a fast algorithm for the direct solution of the integral equation method is presented for essentially 2-D convex scatterers using multilevel non-uniform grids. And in [16], the multilevel matrix decomposition algorithm (MLMDA) is shown to compress LU-decomposed inverse integral operators for analyzing 2-D TM_z scattering phenomena. What's more, the skeletonization process and Huygens' surface are utilized in [17] for a fast direct matrix solver for surface integral equation methods for electromagnetic scattering problems. These state of the art methods

* Corresponding author.
E-mail address: limengmeng@njust.edu.cn (M. Li).

https://doi.org/10.1016/j.enganabound.2017.10.005
Received 19 May 2017; Received in revised form 10 October 2017; Accepted 11 October 2017
0955-7997/© 2017 Elsevier Ltd. All rights reserved.

show highlighted effectiveness in the solution of integral equation methods for their corresponding electromagnetic problems. However, all the above methods are implemented in a sequential fashion because of recursive operations.

In this paper, a novel parallelizable direct solution is proposed with $O(N\log^2 N)$ complexity for electromagnetic analysis in low to intermediate frequency regime. It is directly inspired by the work in [18] where a direct solver is introduced for hierarchical systems. Then [20] applied this method to solve the matrix equation of characteristic basis function method. The main novel contributions of our work in comparison to previous work in [18] and [20] consist of three points. Firstly, a novel parallelizable direct solution of integral equation methods with high parallel efficiency is proposed for electromagnetic analysis. There are mainly two parts of the proposed direct solution: forward decomposition and backward substitution. Both the forward decomposition and backward substitution with $O(N\log^2 N)$ complexity are carried out independently to each group at the same level. Therefore, they can be friendly accelerated with parallelization. The detail of the parallel implementation with high parallel efficiency is proposed in Section 2.4. Secondly, the proposed solution has an advantage on multiscale problems. The RWG basis function [21] is used to discretize the multiscale models without any approximate reduction, which can fit the shape of the multiscale structure precisely. Benefit from the kernel independent property of the proposed solution, an adaptive grouping technique based on both the geometric position and unknowns is proposed to ensure the load balance in each group. The level number of the binary tree is just determined by the number of unknowns in each group at the finest level, which is set to be no greater than a threshold. Thirdly, an effective preconditioner for the iterative solver based on the proposed direct solution is proposed with a selection criterion, which approximates the impedance matrix in an inexpensive way. The selection criterion will be introduced in the Section 2.5 in detail.

2. Theory and implementation

The EFIE formulation for electromagnetic analysis is derived as

$$j\omega\mu\int_S\left[\mathbf{J}(\mathbf{r}')+\frac{1}{k^2}\nabla(\nabla\cdot\mathbf{J}(\mathbf{r}'))\right]G(\mathbf{r},\mathbf{r}')dS=\mathbf{E}^{inc}(\mathbf{r}) \tag{1}$$

Where $\mathbf{J}$ is the induced current on the surface of the object, $\mathbf{E}^{inc}$ is the incident electric field, $\mathbf{r}$ and $\mathbf{r}'$ is the location of the field point and source point, respectively. $G(\mathbf{r}, \mathbf{r}')$ is the Green's function for the electric field due to current sources and is given by

$$G(\mathbf{r},\mathbf{r}')=\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{2}$$

To solve the EFIE in Eq. (1), the induced current $\mathbf{J}$ is discretized with the RWG basis functions $\mathbf{f}(\mathbf{r})$ [21] and the Eq. (1) can be converted into a matrix equation $\mathbf{A}\bullet\mathbf{X}=\mathbf{b}$ with the Galerkin method. The element of the impedance matrix $\mathbf{A}$ and the right side vector $\mathbf{b}$ are expressed as

$$A_{mn}=j\omega\mu\int_{S_m}\int_{S_n}\left[\mathbf{f}_m(\mathbf{r})\cdot\mathbf{f}_n(\mathbf{r}')-\frac{1}{k^2}\nabla\cdot\mathbf{f}_m(\mathbf{r})\nabla\cdot\mathbf{f}_n(\mathbf{r})\right]G(\mathbf{r},\mathbf{r}')dS'dS \tag{3}$$

$$b_m=\int_{S_m}\mathbf{f}_m(\mathbf{r})\cdot\mathbf{E}^{inc}dS \tag{4}$$

2.1. Adaptive grouping technique

The impedance matrix $\mathbf{A}$ for the EFIE is a dense matrix, which can be decomposed into a hierarchical system through the adaptive binary tree grouping technique as shown in Fig. 1. At each level, each parent group is divided into two child groups based on both the geometric position and unknowns to ensure the load balance in each child group. A threshold of unknowns in the group at the finest level is set to determine the level number of the binary tree. Without loss of generality, a two-level hierarchical matrix as an example is shown in Fig. 2. In the hierarchical system, the off-diagonal block matrices $\mathbf{A}_{ij}$ can be approximated by a product of two smaller matrices with matrix decomposition techniques, such as ACA [9,10], ACA-SVD [13], etc., yielding

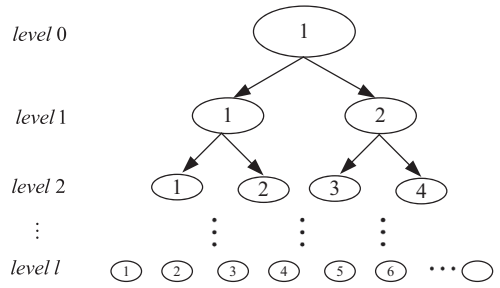

Fig. 1. The adaptive binary tree grouping structure.

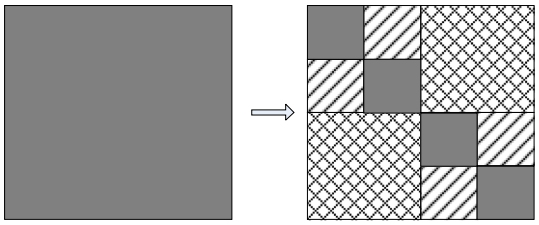

Fig. 2. The hierarchical system of a two-level matrix for a dense impedance matrix.

$$\mathbf{A}_{ij}=\mathbf{U}_{ij}\cdot\mathbf{V}_{ij} \tag{5}$$

i and j are the group number for the interaction groups. The column size of $\mathbf{U}_{ij}$ and the row size of $\mathbf{V}_{ij}$ are decided by the rank of $\mathbf{A}_{ij}$, which is less than the dimension of $\mathbf{A}_{ij}$. Therefore, the impedance matrix $\mathbf{A}$ can be sparsely expressed as

$$\mathbf{A}=\begin{bmatrix}\begin{bmatrix}\mathbf{A}^2_{11} & \mathbf{U}^2_{12}\mathbf{V}^2_{12}\\ \mathbf{U}^2_{21}\mathbf{V}^2_{21} & \mathbf{A}^2_{22}\end{bmatrix} & \mathbf{U}^1_{12}\mathbf{V}^1_{12}\\ \mathbf{U}^1_{21}\mathbf{V}^1_{21} & \begin{bmatrix}\mathbf{A}^2_{33} & \mathbf{U}^2_{34}\mathbf{V}^2_{34}\\ \mathbf{U}^2_{43}\mathbf{V}^2_{43} & \mathbf{A}^2_{44}\end{bmatrix}\end{bmatrix} \tag{6}$$

The superscript in Eq. (6) denotes level number. The subscript is the group number of the interaction groups at the corresponding level.

2.2. The algorithm

For the hierarchical system in Eq. (6) of the impedance matrix, the diagonal block matrices can be extracted. Then the diagonal blocks of the impedance matrix will change into identity matrices, and all the corresponding $\mathbf{U}$ matrices should be updated to $\mathbf{U}'$with an update operation for equality. Therefore, the impedance matrix will be equal to the product of several block diagonal matrices, yielding

$$\begin{bmatrix}\begin{bmatrix}\mathbf{A}^2_{11} & \mathbf{U}^2_{12}\mathbf{V}^2_{12}\\ \mathbf{U}^2_{21}\mathbf{V}^2_{21} & \mathbf{A}^2_{22}\end{bmatrix} & \mathbf{U}^1_{12}\mathbf{V}^1_{12}\\ \mathbf{U}^1_{21}\mathbf{V}^1_{21} & \begin{bmatrix}\mathbf{A}^2_{33} & \mathbf{U}^2_{34}\mathbf{V}^2_{34}\\ \mathbf{U}^2_{43}\mathbf{V}^2_{43} & \mathbf{A}^2_{44}\end{bmatrix}\end{bmatrix}=\mathbf{D}_2\mathbf{D}_1\mathbf{D}_0$$

$$\mathbf{D}_2=\begin{bmatrix}\mathbf{A}^2_{11} & & & \\ & \mathbf{A}^2_{22} & & \\ & & \mathbf{A}^2_{33} & \\ & & & \mathbf{A}^2_{44}\end{bmatrix},\quad \mathbf{D}_1=\begin{bmatrix}\mathbf{A}^1_{11} & \\ & \mathbf{A}^1_{22}\end{bmatrix},$$

$$\mathbf{D}_0=\mathbf{A}^0_{11}=\begin{bmatrix}\mathbf{I} & \mathbf{U}^1_{12}{}'\mathbf{V}^1_{12}\\ \mathbf{U}^1_{21}{}'\mathbf{V}^1_{21} & \mathbf{I}\end{bmatrix} \tag{7}$$

$$\mathbf{A}^l_{k,k}=\begin{bmatrix}\mathbf{I} & \mathbf{U}^{l+1}_{2k-1,2k}{}'\mathbf{V}^{l+1}_{2k-1,2k}\\ \mathbf{U}^{l+1}_{2k,2k-1}{}'\mathbf{V}^{l+1}_{2k,2k-1} & \mathbf{I}\end{bmatrix} \tag{8}$$

Download English Version:

https://daneshyari.com/en/article/6925120

Download Persian Version:

https://daneshyari.com/article/6925120

Daneshyari.com