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Analysis of underwater acoustic scattering problems using stable node-based smoothed finite element method

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ABSTRACT

A stable node-based smoothed finite element method (SNS-FEM) is presented that cures the "overly-soft" property of the original node-based smoothed finite element method for the analysis of underwater acoustic scattering problems. In the SNS-FEM model, the node-based smoothed gradient field is enhanced by additional stabilization term related to the gradient variance items. It is demonstrated that SNS-FEM provides an ideal stiffness of the continuous system and improves the performance of the NS-FEM and FEM. In order to handle the acoustic scattering problems in unbounded domain, the well known Dirichlet-to-Neumann (DtN) boundary condition is combined with the present SNS-FEM to give a SNS-FEM-DtN model for exterior acoustic problems. Several numerical examples are investigated and the results show that the SNS-FEM-DtN model can achieve more accurate solutions compared to the NS-FEM and FEM.

1. Introduction

Acoustic scattering from objects is an interesting physical phenomena and it is of great importance in various practical application such as underwater acoustics, exploration engineering, non-destructive testing and particle manipulation. During the past few decades, a great number of researches have been conducted regarding this problem. Initial works are mainly focused on the objects with particular geometry where separation of variables is applicable. For example, the exact solutions have been obtained for rigid or elastic spherical solids and shells [1–4], infinite cylinders [1], and spheroids [5,6]. Subsequently, there is a variety of new methods have been developed to solve the acoustic scattering problems. These methods include the perturbation method [7], the Green's function method [8], the *T*-matrix method [9], the Fourier matching method (FMM) based on conformal mapping [10], the boundary integral equations [11] and the partial wave series expansions (PWSE) method [12–15]. However, each of these methods has their own associated advantages, disadvantages and conditions of applicability.

Currently, with the fast development of the computer simulation techniques, the standard finite element method (FEM) and boundary element method (BEM) have been the two most popular and powerful numerical methods in coping with the time-harmonic acoustic scattering problems. The classical BEM can be classified as a boundary discretization method and the main advantage of this method is that only boundary discretization is required. In addition, the BEM can naturally satisfy the required Sommerfeld radiation condition at infinity, while some special treatments are needed when the FEM is used. However, the resulting system equations of BEM are usually non-symmetric and dense, which is opposed to symmetric and banded in FEM. This may increase the processing time and storage requirements. Besides, the potential non-uniqueness of the BEM solution at characteristic wave number values is also an important issue. In the contrast to BEM, the FEM, which are based on variational formulations, has a rich mathematical background and the convergence of FEM to the exact solution is well-proved. There is no theoretical limitation on the applicability of FEM to high wave numbers as long as the sufficient refined mesh is used. Recently a meshless boundary collocation method, the singular boundary method (SBM), has been proposed for exterior acoustic problems [16–18]. In SBM, the concept of source intensity factor is introduced to regularize the singularities of fundamental solutions. It successfully overcomes some shortcomings of the original BEM and can be a good alternative for exterior acoustic problems. However, the mathematical theoretical analysis of the SBM seems to be not as complete as the FEM and the relevant work is still on

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its way. In the author's opinion, there is no ideal method has been found yet and the quest for such a method will continue.

In fact, the standard FEM for handling the acoustic scattering still remains two major challenges. The first challenge is how to treat the exterior acoustic problems in unbounded domains effectively. As is known to all, FEM has been well developed for acoustic problems in bounded domains. In general, the application of FEM to unbounded domains involves a domain decomposition by introducing an artificial boundary around the objects. At the artificial boundary, the well-known Sommerfeld radiation condition should be satisfied so that there is no spurious wave reflected from the far field. There are various approaches can be combined with the FEM for the analysis of acoustic scattering in unbounded exterior domains, such as the Dirichlet-to-Neumann (DtN) map developed by Keller and Givoli [19], the recursion in the Atkinson-Wilcox expansion devised by Bayliss et al. [20], and the recent perfectly matched layer approach proposed by Bérenger et al. [21–23]. Among them, the DtN boundary condition devised by Givoli and Keller is an exact non-reflecting boundary condition. It relates the "Dirichlet datum" to the "Neumann datum" with the help of an integral operator M. Although this boundary condition is non-local, it still possesses high computational efficiency and can obtain much more accurate results than those obtained from various approximate local conditions. Therefore, the DtN boundary condition is chosen to cope with the exterior acoustic problems in this paper.

Second, when using the standard FEM for the solution of acoustic problems addressed by Helmholtz equation, one soon is confronted with the well-known dispersion error issue. More importantly, the larger wave number k is, the stronger dispersion error will be. Therefore, the standard FEM can only provide reliable numerical results in the small wave number range, when it comes to large wave number range, the FEM solutions will deteriorate quickly due to the dispersion error issue. Initial FEM researchers used the "rule of thumb" to obtain relatively reliable solutions. In this criterion, a certain fixed number of elements are needed to resolve a wavelength. However, this criterion only works well in the small wave number range. With the increase of the wave number, the numerical dispersion error will increase dramatically even if this criterion is satisfied. In order to deal with this dispersion error effectively, a great number of numerical techniques have been tested with varying degree of success.

Based on the standard Galerkin FEM, the Galerkin/least-squares finite element method (GLS) are proposed to tackle the dispersion error issue [24–26]. In the GLS model, the residuals in least-squares form are added to the standard Galerkin variational equation. The numerical results show that the GLS exhibits superior properties for acoustic problems and provides accurate solutions with relatively low dispersion error. Babuška and his colleagues developed a quasi-stabilized FEM (QSFEM) to solve the Helmholtz equations in two or more space dimensions [27,28]. It is demonstrated that the dispersion error can be controlled by the QSFEM. However, the QSFEM model is very complicated in the general setting. Also based on the standard Galerkin FEM, Franca et al. proposed the residual-free-bubbles (RFB) method for Helmholtz equation [29]. Unfortunately, it is found that the RFB is effective in one dimensions but not in higher dimensions. Furthermore, the high-order finite element method has also been applied for acoustic problems [30] and significant improvements on accuracy are achieved, but higher cost in computation. In addition to the standard finite element method and the extended finite element method mentioned above, the meshfree methods have also been introduced to solve the acoustic problems, such as the element-free Galerkin method (EFGM) [31,32], the multi-resolution reproducing kernel particle method (RKPM) [33], the radial point interpolation method (RPIM) [34] and the meshless Galerkin least-square method (MGLS) [35]. Although the calculation accuracy can be improved to a certain extent with these methods, the dispersion error in general two and three dimensional acoustic problems still cannot be properly eliminated.

As mentioned in reference [36], the approximate discrete model may be the main reason to cause dispersion error. The stiffness of the discretized model obtained from the standard FEM always behaves stiffer than the original model, leading to the so-called numerical dispersion error. So producing a properly "softened" stiffness for the discrete model is much more essential to control the numerical error. Recently, Liu et al. have proposed a series of smoothed finite element methods (S-FEM) which are formulated by incorporating the gradient smoothing techniques of meshfree methods into the existing standard FEM [37–40]. The S-FEMs have been applied to analyze linear elastic solid mechanics and it is found that S-FEMs possess excellent features. Recently, the S-FEMs have been successfully applied to solve acoustic and coupled structural-acoustic problems [41–45]. In the S-FEM family, the node-based smoothed finite element method (NS-FEM) is formulated by performing the gradient smoothing technique over the smoothing domains associated with nodes [46–48]. The numerical results demonstrate that the NS-FEM can provide an upper bound in the strain energy of the exact solution when a reasonably fine mesh is used. However, NS-FEM is temporally instable and cannot be applied to solve the dynamic problems and acoustic problems directly due to its "overly-soft" property. In order to overcome the temporal instability of the NS-FEM, Cui et al. have proposed the stable node-based smoothed finite element method (SNS-FEM) for elasticity problems and acoustic problems [49,50]. In the SNS-FEM, a extra gradient variance item is added to the smoothed gradient field. It is found that SNS-FEM possesses an ideal stiffness of the continuous system and improves the performance of the NS-FEM and FEM. In the present research, the SNS-FEM is used to solve the underwater acoustic scattering problems which are very important in various scientific fields such as linear and nonlinear wave mechanics, underwater technology and ocean acoustics. In this paper, the SNS-FEM is combined with the DtN boundary condition to give a SNS-FEM-DtN model for acoustic scattering problems. Due to the good performance of the SNS-FEM in interior acoustic problems and elasticity problems, it is expected that the SNS-FEM can solve the acoustic scattering problems with very exact solutions.

2. The exterior boundary value problem for the Helmholtz equation

Consider an infinite acoustic problem domain with homogeneous isotropic medium. The acoustic wave satisfies the following reduced wave equation (or the Helmholtz equation).

$$\Delta p + k^2 p = f \tag{1}$$

where p is the acoustic pressure, k is the wave number, Δ is the Laplace operator and f is the acoustic source term.

Assuming that the surface of the obstacle immersed in the unbounded domain can be decomposed into Dirichlet boundary condition Γ_p and Neumann boundary condition Γ_v, where $\Gamma_p \cap \Gamma_v = \varnothing$. The Dirichlet boundary condition and Neumann boundary condition can be described as follows:

$$p = p_D \quad \text{on } \Gamma_p \tag{2}$$

$$\nabla p \cdot n = -j\rho\omega v_n \quad \text{on } \Gamma_v \tag{3}$$

where $j = \sqrt{-1}$, ρ is the density of the medium, ω is the angular