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Three-dimensional fracture propagation with numerical manifold method

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ABSTRACT

By introducing the concept of mathematical cover and physical cover, the numerical manifold method (NMM) is able to solve continuous and discontinuous problems in a unified way. In this paper, the NMM is developed to analyze three dimensional (3D) fracture propagation. The maximum tensile stress criterion is implemented to determine whether the fracture will propagate and the direction of fracture propagation. Three benchmark problems are analyzed to validate the present algorithm and program. The numerical results replicate available experimental results and existing numerical results. The present algorithm and 3D NMM code are promising for 3D fracture propagation. They deserve to be further developed for the analysis of rock mechanic problems in which the initiation and propagation of multiple fractures, tensile and shear fractures, and fracture propagation under compressive loading are taken into account.

1. Introduction

Analyzing the evolution of fractures in fractured rock mass is of great importance in many fields [1]. With the development of computer science, the numerical method has become one of the most effective approaches to understand fracture evolution, which attracts a great number of researchers during the last decade. Up to now, a great number of versatile numerical approaches have been proposed to simulate fractures, such as Finite Element Method (FEM) [2] with remeshing strategy, Boundary Element Method (BEM) [3] and meshfree methods [4].

Nowadays, Finite Element Method (FEM) [2] is the most widely used numerical approach in engineering and have been utilized to simulate three-dimensional (3D) fracture propagation for several decades. However, it still suffers from the significant difficulties in mesh generation and refinement during fracture simulation. In FEM, fracture surfaces must coincide with the element boundaries, therefore the meshes must be updated at each simulation step. Additionally, the meshes are required to be more refined in the vicinity of fracture tips than in the remainder of the model, in order to obtain sufficiently accurate solution for the fracture analysis [5]. Especially, when the problems are taken into account in 3D, the simulations are significantly more difficult. Polygonal Finite Element Method (PFEM) [6–8] is a development of FEM, which is able to construct proper approximations on polygonal elements, and provides an effective approach to remesh and refinement in two dimensions [9]. In order to model the development of arbitrary multiple fractures in 3D, Paluszny has developed a robust simulator using global remeshing strategy [10], which has been successfully utilized in investigating block caving system [11–13], oil recovery [14,15], and predicting the permeability of three-dimensional fractured porous rock [16]. However, global strategy is not suitable in some special problems. When the fractured zone is obviously smaller than the whole computational model, this global strategy is not cost-effective. Therefore, a number of methodologies are developed for introducing fractures into computational model without remeshing, including the numerical manifold method (NMM) which is developed in this work.

The Boundary Element Method (BEM) is an alternative approach to solve fracture propagation problems because it could reduce the dimensions of the problems and simplify the complexity [17]. In 2014, Wu and Olson [18] utilized BEM to study the simultaneous multiple fracture treatments in hydraulic fracturing. The main drawbacks of BEM are the difficulty to solve nonlinear problems [19] and the difficulty to handle computational models which contains many different materials.

Meshfree methods do not need a mesh to discretize the problem domain, and therefore are very suitable to solve complex

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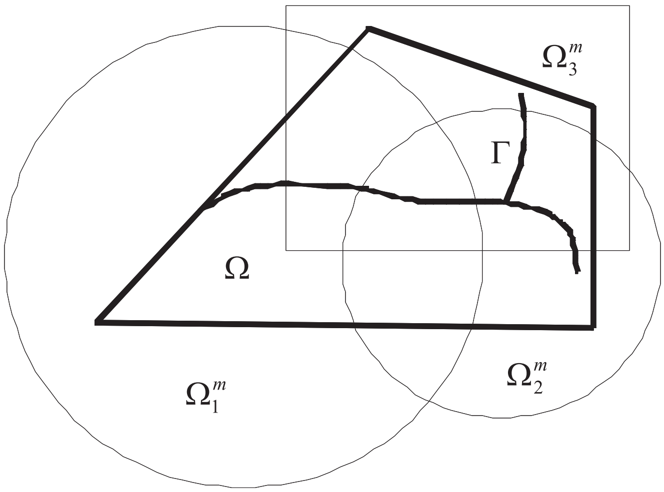


Fig. 1. Problem domain (thick lines) and mathematical cover (fine lines) [40].

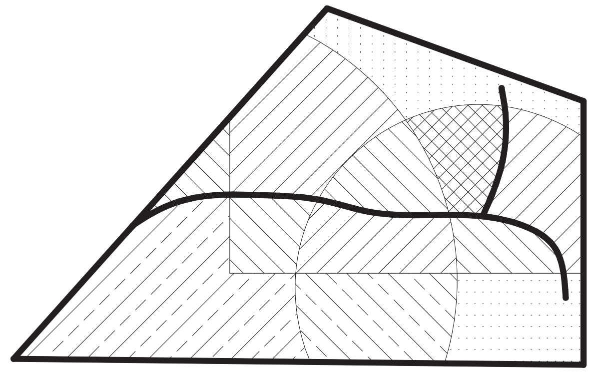
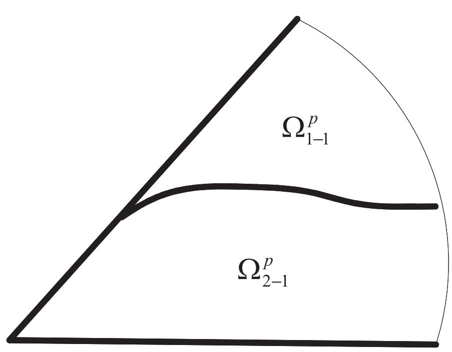


Fig. 3. Manifold elements from the PC [40].

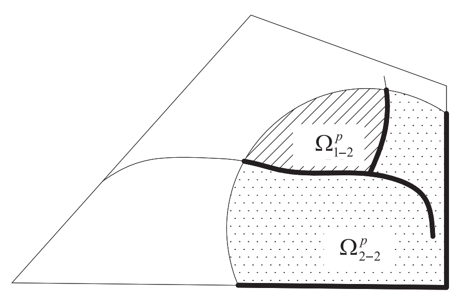
practical problems such as large deformation [20] and fracture propagation simulation [21]. The main contributions to the development of meshfree methods are known in the literatures as Element-Free Galerkin method (EFG) [22], Reproducing Kernel Particle Method (RKPM) [23] and stable particle methods [24]. Bordas et al. [25] have shown that the high flexibility of meshfree methods can be exploited to model arbitrary three-dimensional fracture initiation, propagation, branching and junction in non-linear materials. Nonetheless the high computational cost and complex process in constructing the trial functions will deteriorate the stability and efficiency of numerical integration [26]. Moreover, they cannot be implemented into existing finite element data structures [19].

In order to overcome the burden of meshing and remeshing of the FEM in modeling three-dimensional fracture problems, some Partition of Unity [27] based approaches have been developed, which can be considered as an improvement of FEM. Typical of them are the eXtended Finite Element Method (XFEM) [28,29] and generalized finite element method (GFEM) [19,30]. In XFEM, the generalized Heaviside functions and the asymptotic fracture-tip functions are incorporated into the FEM to account for the fractures, without the need for the finite element mesh to conform to the fractures [31]. In GFEM [30], the standard finite element spaces are augmented by adding special functions which reflect the known information about the boundary value problem and the input data to model problems with multiple straight reentrant corners, voids, and fractures.

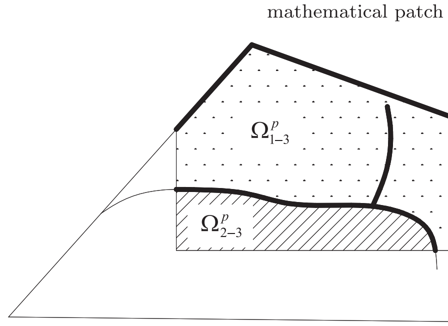
In 1991, Shi [32] developed numerical manifold method (NMM) for geotechnical engineering, which also falls into the category of the partition of unity. The main attractive advantage of NMM is to



(a) Physical patches Ω_{1-1}^p and Ω_{2-1}^p generated from mathematical patch Ω_1^m



(b) Physical patches Ω_{1-2}^p and Ω_{2-2}^p generated from mathematical patch Ω_2^m



(c) Physical patches Ω_{1-3}^p and Ω_{2-3}^p generated from mathematical patch Ω_3^m

Fig. 2. Physical patches from mathematical patches [40], (a) Physical patches Ω_{1-1}^p and Ω_{2-1}^p generated from mathematical patch Ω_1^m (b) Physical patches Ω_{1-2}^p and Ω_{2-2}^p generated from mathematical patch Ω_2^m (c) Physical patches Ω_{1-3}^p and Ω_{2-3}^p generated from mathematical patch Ω_3^m.