Engineering Analysis with Boundary Elements 38 (2014) 8–16

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Extended displacement discontinuity method for nonlinear analysis of penny-shaped cracks in three-dimensional piezoelectric media

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ARTICLE INFO

Article history:
Received 22 July 2013
Accepted 29 September 2013
Available online 31 October 2013

Keywords:
Piezoelectric medium
Penny-shaped crack
Non-linear fracture
Polarization saturation (PS) model
Dielectric breakdown (DB) model
Extended displacement discontinuity method

ABSTRACT

The polarization saturation (PS) model and the dielectric breakdown (DB) model are both used, under the electrically impermeable crack assumption, to analyze penny-shaped cracks in the isotropic plane of three-dimensional (3D) infinite piezoelectric solids. Using the extended displacement discontinuity integral equation method, we obtained analytical solutions for the size of the electric yielding zone, the extended displacement discontinuities, the extended field intensity factor and the *J*-integral. Integrating the Green function for the point extended displacement discontinuity provided constant element fundamental solutions. These solutions correspond to an annular crack element applied with uniformly distributed extended displacement discontinuities in the transversely isotropic plane of a 3D piezoelectric medium. Using the obtained Green functions, the extended displacement discontinuity boundary element method was developed to analyze the PS model and DB model for penny-shaped cracks. The numerical method was validated by the analytical solution. Both the analytical results and numerical results show that the PS and the DB models give equivalent solutions for nonlinear fracture analysis of 3D piezoelectric materials, even though they are based on two physically different grounds.

1. Introduction

Piezoelectric materials are widely used in smart structures and engineering because of their mechanical-electric coupling effects. Nonlinear fractures of piezoelectric media under mechanical and electric loadings have attracted a lot of attention [1–3]. For conventional elastic materials, the Dugdale model [4] is one of the most famous and important models for describing nonlinear fracture mechanics. This model assumes that the elastic–plastic material is ideal, and that the stress in the yielding zone is constant and equal to the yield value. Fracture mechanics based on the Dugdale model have been studied extensively and have been applied to two-dimensional (2D) [5–11], as well as three-dimensional (3D) situation [12–14]. Extending the Dugdale model to piezoelectric materials, Gao and Barnett [15] and Gao et al. [16] proposed a strip polarization saturation (PS) model in which the piezoelectric material is treated as mechanically brittle and electrically ductile and the electric displacement is equal to the polarization saturation value in the electric yielding zone. Intensive studies of this model have been applied to 2D media by numerous researchers [17–28]. In 1999, Zhao et al. [29] derived the electric yielding zone in the PS solution for a penny-shaped crack in 3D transversely isotropic piezoelectric media. McMeeking also studied the PS model and pointed out that the electric field strength of the PS model should correspond to the stress determined by the classical Dugdale model from an energy perspective [30]. For this reason, Zhang and Gao proposed the strip dielectric breakdown (DB) model [31,32], in which the electric field strength in the electric yielding zone reaches the dielectric breakdown strength. It was found that the PS and DB models are qualitatively equivalent to each other for the fracture analysis except for a slight difference between their *J*-integral values. The DB model has since been further investigated [2,33,34]. However, nonlinear fracture studies for 3D piezoelectric materials are still limited to the PS model without a full solution and without any solution on the DB model.

It is usually difficult to obtain analytical solutions to problems involving finite solids or complex boundary conditions. The numerical method provides an alternative for studying such problems. Numerical analyses of Dugdale model for 2D and 3D purely elastic–plastic materials have been conducted [10,11,13,35–37]. Bhargava and Kumar numerically studied a strip yield model under anti-plane shear stress and in-plane electric loading for crack arresting of a piezoelectric strip [38]. Linder analyzed the exponential electric displacement saturation model in fracturing piezoelectric ceramics [39]. Fan et al. studied both the PS and DB models via a non-linear hybrid extended displacement discontinuity-fundamental solution

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http://dx.doi.org/10.1016/j.enganabound.2013.09.014

(NLHEDD-FS) method under both electrically impermeable and semi-permeable conditions [40]. Very recently, Fan et al. extended the NLHEDD-FS method to the PS model analysis of an arbitrarily oriented crack in a finite 2D piezoelectric medium [41].

Motivated by the above results, in this paper, we derive analytical solutions for both the PS and DB models for a penny-shaped crack in 3D piezoelectric media via the extended displacement discontinuity integral equation method and develop an extended displacement discontinuity boundary element method. The paper is organized as follows: in Section 2, we will derive the analytical solutions for the two models; in Section 3, numerical methods will be presented and numerical results will be presented. Conclusions will be drawn in Section 4.

2. Analytical solution for nonlinear models of penny-shaped cracks

A penny-shaped crack S of radius a is located in the isotropic plane of a 3D piezoelectric medium. The polarizing direction is in the z-axis of the rectangular coordinates $OXYZ$ and centered at the origin O. This schematic is shown in Fig. 1. Uniformly distributed mechanical loading p_0 and electric loading ω_0 with the same values but in opposite directions are applied to the upper and lower surfaces

$$\begin{array}{l} p(X,Y)=p_0, \\ \omega(X,Y)=\omega_0, \end{array} \quad 0\le r\le a,\ X=r\cos\theta,\ Y=r\sin\theta,\ 0\le\theta\le 2\pi, \tag{1a}$$

where $Or\theta Z$ is the cylindrical coordinate system corresponding to the rectangular system of $OXYZ$. Based on the geometry and the applied loadings, it can be seen that the problem is axisymmetric. Thus, the solution is a function of the coordinate r and Z, which for the plane $Z=0$ depends on the coordinate r only. Eq. (1a) can be rewritten as

$$\begin{array}{l} p(r)=p_0, \\ \omega(r)=\omega_0, \end{array} \quad 0\le r\le a. \tag{1b}$$

Based on the nonlinear models for piezoelectric media [16,33], we can assume that the electric yielding zone surrounding the penny-shaped crack is an annular region in the plane $Z=0$. In the electric yielding zone $S_c=\{a\le r\le c;\ \ 0\le\theta\le 2\pi\}$ for the PS model, the piezoelectric displacement D_z reaches its saturated value D_s (see Fig. 2a)

$$\omega(r)=-D_s,\quad a<r\le c. \tag{2}$$

Meanwhile in the electric yielding zone $S_b=\{a\le r\le b;\ \ 0\le\theta\le 2\pi\}$ for the DB model, the electric strength E_z reaches its breakdown value E_b (see Fig. 2b)

$$E(r)=E_b,\quad a<r\le b. \tag{3}$$

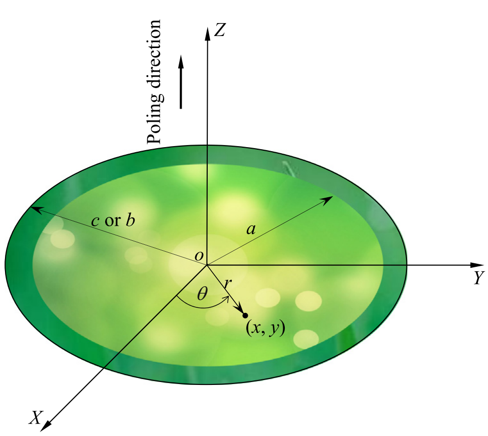

Fig. 1. A penny-shaped Dugdale crack of radius a in the transversely isotropic plane of an infinite piezoelectric medium, with the electric yielding zone $a<r<c$ for the PS model or $a<r<b$ for the DB model.

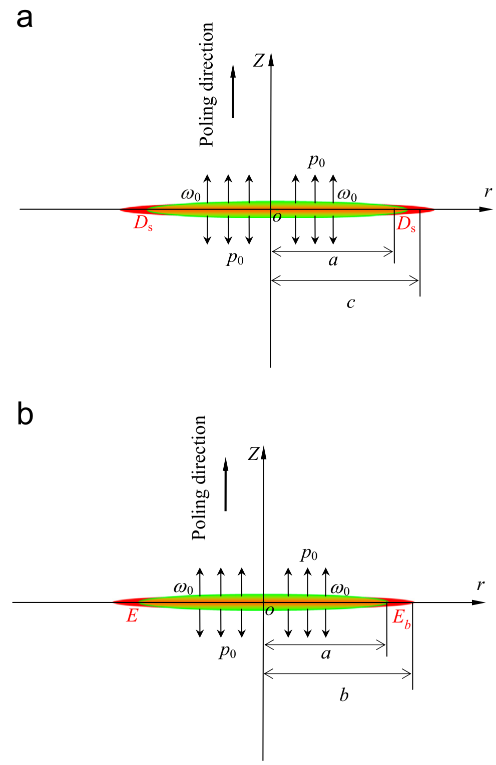

Fig. 2. (a) The PS model of a penny-shaped crack under applied loadings on its surface. (b) The DB model of a penny-shaped crack under applied loadings on its surface.

2.1. Solution of the PS model

The extended displacement discontinuity integral equations are given as [42]

$$\int_S L_{31}||w||\frac{1}{R^3}dS(\xi,\eta)+\int_{S_c}L_{32}||\varphi||\frac{1}{R^3}dS(\xi,\eta)=-p(X,Y),(X,Y)\in S, \tag{4}$$

$$\int_S L_{41}||w||\frac{1}{R^3}dS(\xi,\eta)+\int_{S_c}L_{42}||\varphi||\frac{1}{R^3}dS(\xi,\eta)=-\omega(X,Y),(X,Y)\in(S+S_c), \tag{5}$$

where the material-related constants L_{ij} are given in Appendix A and the elastic displacement discontinuity $||w||$ and the electric potential discontinuity $||\varphi||$ in the z-direction are defined as

$$\begin{array}{l} ||w(r)||=w(r,Z^+)-w(r,Z^-), \\ ||\varphi(r)||=\varphi(r,Z^+)-\varphi(r,Z^-), \end{array} \quad Z=0, \tag{6}$$

and

$$R=\sqrt{(X-\xi)^2+(Y-\eta)^2}. \tag{7}$$

The axisymmetry allows Eq. (5) to be transformed into the Abel type integral equation [29,43]

$$4\int_0^r\frac{t^2}{(r^2+t^2)^{3/2}}\left[\int_t^c\frac{\rho(L_{41}||w||+L_{42}||\varphi||)}{(\rho^2-t^2)^{3/2}}d\rho\right]dt=\omega(r),\quad 0\le r\le c, \tag{8}$$

and its solution can be expressed by

$$L_{41}||w||+L_{42}||\varphi||=L_{42}f_\phi(r),\quad 0<r<c, \tag{9}$$