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Return mapping algorithm in principal space for general isotropic elastoplasticity involving multi-surface plasticity and combined isotropic-kinematic hardening within finite deformation framework

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ABSTRACT

The compatibility with complicated elastoplasticity and efficiency of the constitutive integration algorithm both significantly influence the performance of finite element analysis for engineering practical problems. In this work, a numerical integration algorithm in principal space is proposed for general isotropic elastoplastic constitutive models that involve multi-surface plasticity with corners in the yield surface and combined isotropic-kinematic hardening law as well as nonlinear elasticity within the framework of finite deformation. For the multi-surface plasticity, a strategy, which uses the mid-direction of two plastic flow directions at a corner as the border of critical regions, is proposed to predict the yield functions activated in the return mapping iterations, making the prediction procedure simpler. By making use of the relative stress, the combined isotropic-kinematic hardening law is incorporated into the numerical integration algorithm in principal space. The consistent tangent operator is also derived. Besides, the fully implicit return mapping algorithm based on representation theorem is employed. The expressions of the first and second derivatives of yield/potential function, which are frequently evaluated in the algorithm, maintain a simple form and reduce the computational cost. Solution of finite element practical problems demonstrates that compatibility and efficiency of the constitutive integration algorithm are improved while accuracy is retained.

1. Introduction

The compatibility and efficiency of constitutive integration algorithm in the finite element analysis are both important for scientific inquiry and engineering practice [1,2]. In engineering practice, many materials, such as concrete, soils and rocks, exhibit complex nonlinear mechanical behaviors, including porous elasticity, different yield strength in extension and compression, pressure-sensitive yielding, the Bauschinger effect, etc. To capture these behaviors, complicated elastoplastic constitutive models are usually used, which often involve nonlinear elasticity, multi-surface plasticity with corners/singularities in yield/potential function [3–5] and combined isotropic-kinematic hardening law [6,7]. For isotropic materials, these constitutive models usually involve all the three stress invariants in the yield/potential function [8–10]. Besides, engineering materials often undergo very large deformation or large strain, where the finite strain elastoplastic constitutive models are typically utilized [11,12]. It is obvious that in the case of advanced constitutive models involving nonlinear elasticity, multi-surface plasticity and combined hardening law when implemented in large-scale finite element analyses, the formulation can be a challenging task and the solution of constitutive integration can be rather computationally intensive. Therefore, a fast constitutive integration algorithm, which is compatible with complicated elastoplasticity within the finite deformation framework, is in demand to improve the performance of finite element analysis for engineering practical problems.

In the finite element analysis, integrating the constitutive equations to update the stresses and the internal variables is of crucial importance. Since the constitutive integration needs to be carried out at all yielded integration points for each load increment and each equilibrium iteration, the constitutive integration algorithm sometimes predominantly influences the overall accuracy and efficiency of the finite element solution [1,2]. Moreover, the constitutive integration algorithm determines the formulation of consistent tangent operator which is important for attaining the quadratic rate of convergence in the global Newton iterations [1,13,14]. However, as mentioned earlier, many engineering materials exhibit complex mechanical behaviors and thus lead to

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complicated constitutive models. In these circumstances, constitutive equations can not be integrated analytically to obtain a closed form and thus the numerical integration techniques are usually employed. Among those numerical integration techniques, the return mapping algorithm based on the fully implicit backward Euler difference scheme has been widely accepted because of its unconditional stability and excellent robustness [14–16]. An overview of the return mapping algorithm can be found in the book by Crisfield [4]. Obviously, the compatibility with complicated elastoplasticity and efficiency of the integration algorithms are the key to improve the stress update procedure and finite element solution. To this end, a number of numerical integration algorithms formulated in principal space have been proposed in the literature for the last decades [5,9,10,17–23]. These numerical integration algorithms in principal space can be classified into three categories:

(1) Algorithms based on space transformation [5,17,18]. Following Simo's approach, some authors have carried out the numerical integration in principal stress space and subsequently transformed it back to the general six-dimensional stress space. In this way, numerical determination of the principal directions and the principal values is required. For instance, Larsson and Runesson [18] established an implicit integration algorithm and consistent tangent operator for yield criteria of the Mohr-Coulomb type in principal stress space. Their integration algorithm, which only considers isotropic hardening, has been applied to multi-surface plasticity and covered all the possible cases of regular, corner and apex solutions. In a later study, Peric and Neto [5] proposed a numerical integration scheme in principal stress space for the rate-independent elastoplastic models with yield surfaces containing corners/singularities and general nonlinear isotropic hardening. By employing the multiplicative plasticity and logarithmic strain measures [2,12], they extended their algorithm to finite strains. Although, in this kind of algorithm, the problem can be reduced to three dimensions by using the principal values, the determination of the principal directions/values and the transformation of results between the principal and general spaces are still needed and computationally expensive.

(2) Algorithms based on spectral decomposition [9,10,19,20]. This kind of algorithm refers to the eigenvalues and eigenvectors of the stress or stain tensor, which are essentially the principal values and principal directions of the tensor. Since an intrinsic and full tensorial description is used in the global coordinate system, this algorithm is completed by matrix addition rather than matrix multiplications as in algorithm 1, leading to less mathematical operations and more efficient. For instance, Tamagnini et al. [9] and Borja et al. [10] both employed this kind of algorithm to formulate the return mapping algorithm and consistent tangent operator for a three-invariant isotropic hardening elastoplastic model. In contrast, Borja et al. [10] developed their algorithm within the framework of finite deformation. From a somewhat different approach, Rosati and Valoroso [19] developed an algorithm starting from the derivatives of the eigenvalues and eigenvectors of a symmetric second-order tensors with respect to the tensor itself. On the other hand, Foster et al. [20] modified the algorithm by using the spectral decomposition of the relative stress in order to incorporate the kinematic hardening into the integration procedure within the infinitesimal framework. Indeed, spectral decomposition can avoid space transformation procedure. However, the functions and equations derived in this algorithm are complicated and the numerical determination of the principal directions or eigenvectors is still required as usual.

(3) Algorithms based on representation theorem [21–23]. In this kind of algorithm, a set of three base tensors in conjunction with a set of three invariants are employed for the representation of stress or strain tensor. The base tensors, which are directly constructed by zero-order, first-order and second-order power of an argument second-order tensor, are complete and irreducible bases according to the representation theorem [24,25]. By using the based tensors and corresponding invariants, the return mapping algorithm can be formulated in the three-dimensional principal space. The transformation procedure between the principal and general spaces can be avoided. Moreover, the computation of the principal directions can be bypassed. For instance, Palazzo et al. [21] described an integration strategy by using base tensors for infinitesimal three-invariant elastoplastic models with combined isotropic-kinematic hardening law. Their base tensors are composed of the second-order identity tensor, the stress deviator and its square, which are non-orthogonal in principal space. As pointed out by Criscione et al. [26], non-orthogonality leads to constitutive models that are ill-suited for fitting parameters to experimental data because they yield highly covariant response terms. Moreover, the utilization of non-orthogonal base tensors will lead to complicated and lengthy tensor and matrix operations, reducing the efficiency of the algorithm. This drawback was later improved by Peng and Chen [22], who developed a return mapping algorithm for isotropic hardening constitutive model by employing a set of three mutually orthogonal unit base tensors. Their base tensors are composed of a normalized identity tensor and two additional unit deviatoric tensors taking 0 and $\pi/2$ Lode angles, respectively. By virtue of the mutually orthogonal unit base tensors and corresponding invariants, the formulas associated with return mapping iteration and the consistent tangent operator can be derived in a simple form, making the algorithm more efficient. More recently, Huang et al. [23] further extended the work of Peng and Chen [22] to include the finite strain effects for large deformation analysis.

Aforementioned literature review shows different return mapping algorithms in principal space to improve the efficiency of the constitutive integration algorithm. In addition to the efficiency, these algorithms also focus on complex mechanical behaviors and thus consider complicated constitutive models (e.g. involve multi-surface plasticity, isotropic and/or kinematic hardening law, finite strain/deformation) in order to improve compatibility of the constitutive integration algorithm with complicated elastoplasticity. However, to the best of our knowledge, a return mapping algorithm in principal space, which can both consider multi-surface plasticity and combined isotropic-kinematic hardening law as well as nonlinear elasticity within the framework of finite deformation, has not yet been reported in the literature. In the this work, we are going to develop a compatible and efficient constitutive integration algorithm in principal space based on representation theorem for general isotropic elastoplastic constitutive models that involve nonlinear elasticity, multi-surface plasticity and combined isotropic-kinematic hardening law within the framework of the finite deformation. The rest of the paper is organized as follows. Section 2 briefly describes the base tensors and associated basic operations. Section 3 elaborates the proposed return mapping algorithm involving multi-surface plasticity within the finite deformation framework. Section 4 elaborates the proposed return mapping algorithm involving combined isotropic-kinematic hardening law within the finite deformation framework. Section 5 gives a step-by-step description of the proposed return mapping algorithm involving both multi-surface plasticity and combined hardening law as well as nonlinear elasticity within the finite deformation framework. Section 6 carries out numerical examples of finite element practical problems by implementing the proposed constitutive integration algorithm. Section 7 discusses the cause of efficiency improvement by the proposed algorithm. Finally, Section 8 summarizes conclusions obtained from this research work. In addition, the tensor and matrix notations used in this paper are detailed in Appendix.