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Solution of hyperbolic bioheat conduction models based on adaptive time integrators

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ABSTRACT

In this work, a new time marching technique is proposed to analyze hyperbolic bioheat transfer problems. In this new approach, time integration parameters adapt themselves along the solution process, in accordance to the properties and results of the model. Thus, the time integrators are locally evaluated, assuming different values along the spatial and temporal discretizations, enabling a more accurate and effective solution algorithm. The proposed technique has guaranteed stability, it is truly self-starting, and it is formulated as a non-iterative single-step/solver procedure, demanding low computational efforts. As illustrated in the manuscript, the methodology is very accurate, robust and simple to implement, providing a suitable numerical approach to analyze hyperbolic bioheat conduction models.

1. Introduction

Numerical methods have been widely used for the solution of many problems in the thermo-biology field. Different mathematical models can be used to describe the bioheat transfer process in living tissue [1,2]. The literature reports a substantial number of papers on the numerical modelling of the parabolic Pennes equation considering different numerical techniques [1,3–6]. However, there are some applications involving extremely short time duration or very low temperature (e.g. cryogenic surgery, laser induced thermal damage, etc.) for which the parabolic bioheat equation, which assumes an infinite thermal speed of propagation according to Fourier's law, is not adequate and the mathematical model may be more accurately described by the hyperbolic bioheat equation [7–13]. The hyperbolic bioheat equation is characterised by a finite thermal speed of propagation of the thermal waves due to the application of a modified Fourier's law [14,15], involving a relaxation time τ_q which indicates that there is a delay between the heat flux vector and the temperature gradient. For the same point in the conduction medium, the temperature gradient is established at time t, but the heat flux vector will be established at a later time $t + \tau_q$. Another hyperbolic bioheat equation was developed by Tzou [16], giving origin to the dual phase lag (DPL) model involving two time delays τ_q and τ_T (phase lags), which allows for the heat flux to respond to the temperature gradient or vice-versa, depending on the relative values of the phase lags.

The work presented in this paper proposes a new methodology to analyze hyperbolic bioheat transfer problems, considering the more generic context that is represented by the DPL model. Here, the spatial discretization of the body is carried out taking into account the standard Finite Element Method (FEM) [17,18], and the time domain analysis is carried out taking into account a modified, extended new version of the adaptive methodology proposed by Soares [19], which was developed for dynamic applications. Following this new approach, two time integration parameters are considered, namely α and γ, which are allowed to assume different values at each FEM element and at each time step. The computation of the γ parameter is designed to improve accuracy and to ensure the stability of the analysis. The evaluation of the α parameter, on the other hand, focuses on enabling an effective numerical dissipative algorithm, aiming to eliminate the influence of spurious modes and to reduce amplitude decay errors; it defines the so-called dissipative and non-dissipative elements of the model, which are relabeled at each time step of the analysis. The proposed adaptive strategy is non-iterative, and the values for the time integrators are simply and directly computed taking into account just the physical/geometrical properties of the finite elements of the spatial discretization, the adopted time-step, and local previous time-step results. In addition, the proposed technique is only based on single-step relations involving two variables: the temperature field and its first time derivative. Thus, just a single set of equations has to be dealt with within a time-step, and the resulting method stands as truly

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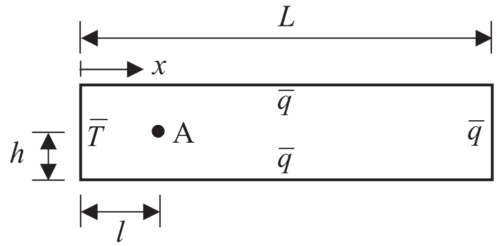

Fig. 1. Sketch of the first model: geometric configurations and boundary conditions.

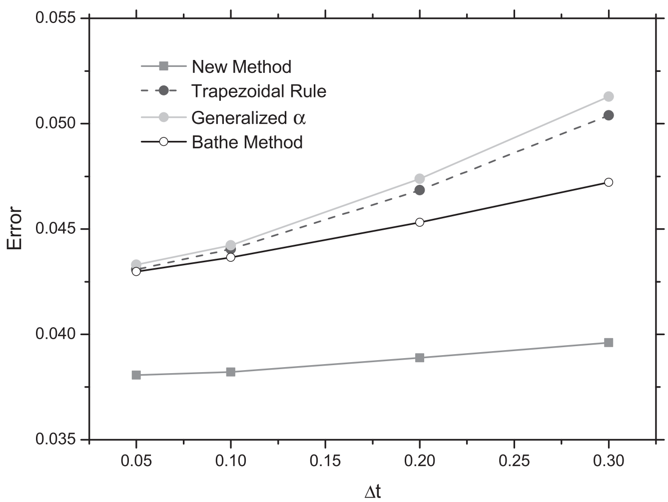

Fig. 2. Error results at point A.

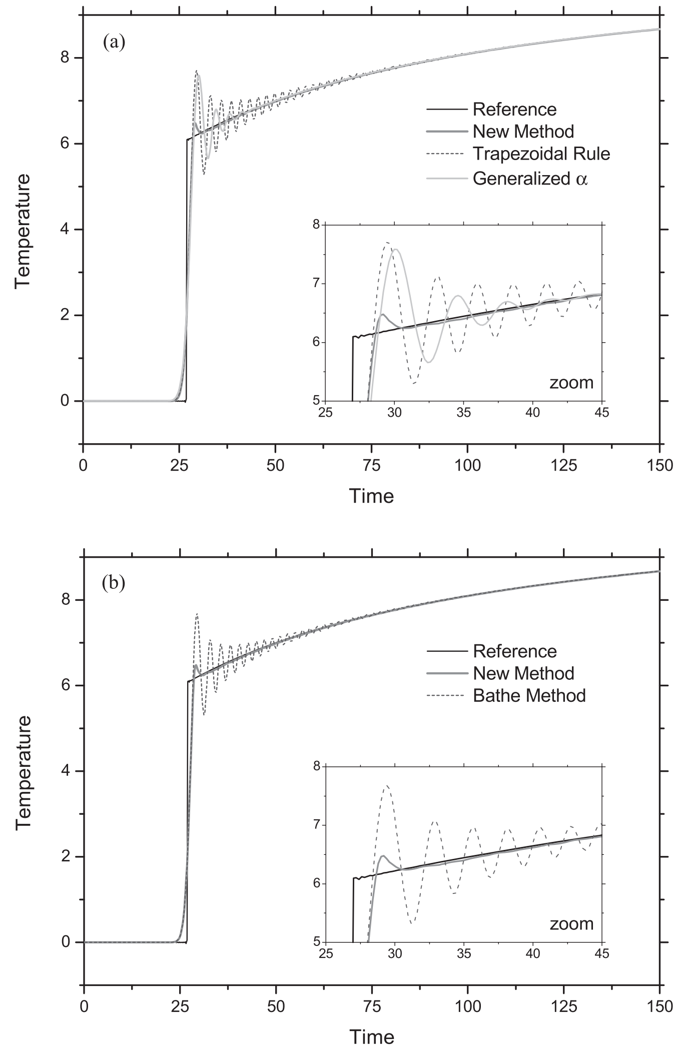

Fig. 3. Time history results at point A, for $\triangle t = 0.2s$: (a) comparison with single-step solution techniques; (b) comparison with multi-step solution techniques.

self-starting, eliminating any kind of cumbersome initial procedure, such as the computation of initial second time derivative values and/or the computation of multistep initial values.

The manuscript is organized as follows: initially, the governing equations of the DPL model are briefly presented and, in the sequence, the adopted spatial and temporal numerical discretizations are discussed, followed by a detailed description of the proposed adaptive technique. Relevant numerical validation tests are then considered, illustrating the accuracy and effectiveness of the proposed methodology. Further details on the mathematical formulation of the new approach are provided in the appendix, where the stability and dissipative features of the technique are discussed in more detail.

2. Governing equations

The dual-phase-lag (DPL) model may be expressed as:

$$\mathbf{q}(\mathbf{x}, t + \tau_q) = -k\nabla T(\mathbf{x}, t + \tau_T) \tag{1}$$

which is a conduction law that allows either the temperature (T) gradient to precede the heat flux vector ($\mathbf{q}$), when $\tau_q > \tau_T$, or the heat flux vector to precede the temperature gradient, when $\tau_q < \tau_T$; where τ_q and τ_T stand for the phase lags for the heat flux vector and temperature gradient, respectively. For $\tau_T = 0$, the modified Fourier's law reduces to the Cattaneo–Vernotte (CV) model [14,15] and, for $\tau_T = 0$ and $\tau_q = 0$, the standard Fourier's law is obtained [20].

Taking into account the DPL model and first-order expansions, the governing equation for hyperbolic bioheat transfer problems reads:

$$\begin{aligned} &\tau_q \rho c \ddot{T}(\mathbf{x}, t) + (\rho c + \tau_q w_b c_b)\dot{T}(\mathbf{x}, t) - \tau_T \nabla \cdot k \nabla \dot{T}(\mathbf{x}, t) = \\ &= \nabla \cdot k \nabla T(\mathbf{x}, t) - w_b c_b (T(\mathbf{x}, t) - T_b(\mathbf{x}, t)) + q'(\mathbf{x}, t) \end{aligned} \tag{2}$$

where over dots indicate time derivatives; k, ρ and c stand for the thermal conductivity, density and specific heat of the tissue, respectively; c_b, w_b and T_b are the specific heat, perfusion rate and temperature of blood, respectively; and the volumetric heat q' contains the metabolic and spatial heating terms q_{met} and q_{ext}, as well as their time derivatives: $q'(\mathbf{x}, t) = q_{met}(\mathbf{x}, t) + q_{ext}(\mathbf{x}, t) + \tau_q(\dot{q}_{met}(\mathbf{x}, t) + \dot{q}_{ext}(\mathbf{x}, t))$.

Once the governing differential equation is established, boundary and initial conditions must be defined. Here, they are given by:

(i) Boundary conditions ($t \geq 0$, $\mathbf{x} \in \Gamma$ where $\Gamma = \Gamma_T \cup \Gamma_q$):

$$T(\mathbf{x}, t) = \overline{T}(\mathbf{x}, t) \text{ for } \mathbf{x} \in \Gamma_T \tag{3a}$$

$$k\nabla T(\mathbf{x}, t) \cdot \mathbf{n} = \overline{q}(\mathbf{x}, t) \text{ for } \mathbf{x} \in \Gamma_q \tag{3b}$$

(ii) Initial conditions ($t = 0$, $\mathbf{x} \in \Omega$):

$$T(\mathbf{x}, 0) = \overline{T}_0(\mathbf{x}) \tag{4a}$$

$$\dot{T}(\mathbf{x}, 0) = \dot{\overline{T}}_0(\mathbf{x}) \tag{4b}$$