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A Bloch wave reduction scheme for ultrafast band diagram and dynamic response computation in periodic structures

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ABSTRACT

A variety of Reduced-Order Modelling (ROM) techniques have been developed for the Wave/Finite Element (WFE) Framework. However most of these techniques are not compatible with dynamic response computation or frequency-dependent problems. This paper introduces a new reduction strategy for the WFE method, enabling the computation of both the forced response and the complex dispersion curves of periodic structures modelled using large-sized finite element (FE) models. The method exploits the duality between Inverse and Direct Bloch formulations to build a reduced solution subspace, accounting for both propagating and evanescent behaviours, while ensuring high reduction factors. This reduction strategy therefore enables the resolution of a wider range of problems, including near/far field response computation in finite waveguides subjected to dynamic loadings, or vibroacoustic transmission/reflection problems. First, the method is used to compute dispersion curves and forced response in a duct. Then a large bi-stiffened structure is studied to evaluate the method's performances. The high frequency resolution provided by the proposed ROM allows us to explore a variety of propagation and guided resonances localization effects, hardly accessible otherwise. Furthermore, the considerable reduction factors enable fast wave dispersion analyses in large-scaled periodic structures, complex phononic crystals designs or locally resonant metamaterials.

1. Introduction

An extensive research effort has been devoted to understand and analyse wave propagation in periodic structures, meta-materials and a broad range of lightweight structures over the past decade. Their periodicity allows the computation of local wave dispersion characteristics such as stop-bands, local resonances, diffusion and spatial attenuation properties. This knowledge can then be used for the design and optimization of a broad range of engineered media with desirable dynamic behaviours (see Refs. [1–4]). Various methods were developed to predict wave dispersion characteristics in complex 1D or 2D periodic structures: homogenization techniques were proposed in Ref. [5] to compute high-frequency dispersion characteristics in phononic waveguides. One can also cite the work Boutin et al. [6] on equivalent models for porous waveguides with embedded Helmoltz resonators. Multi-scale techniques were also developed [7,8] to model heterogeneous or periodic structures using limited macroscopic information. Recently, the wave finite element method (WFEM) based on Bloch theorem, has been the subject of high interest (see Section 2 for a detailed discussion). It has been used to study wave propagation and conduct vibroacoustic analyses in a wide range of continuous or periodic structures, as it exploits standard FE packages to model the waveguide's unit-cell. One can cite applications of the WFEM to structures such as sandwich panels involving different core topologies [9,10], poroelastic media [11] or piezoelectric elements [2]. It was recently applied to perform fast design of periodic topologies with enhanced acoustic performances by computing local acoustic radiation [12] or transmission [13,14] problems in a variety periodic structures.

In order to further enable the development of novel acoustic or elastic meta-structures with optimized structural configurations, considerable challenges are still to be faced in terms of numerical methodologies. Indeed, computing the broadband dispersion diagram of complex periodic waveguides requires to solve numerous large, often ill-conditioned (see Waki et al. [15]) eigenvalue problems, whose size is related to the FE model used to describe the waveguide's unit-cell.

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Remarkable achievements have been achieved on reduced-order modelling (ROM) strategies for wave-based methods and particularly for the wave finite element framework. Those are driven by the need to use refined unit-cell's finite element model when the wavelength are small compared with the unit-cell's length. Reduction schemes are also crucial to enable fast design of complex structures (e.g. to create locally resonant behaviours, see Ref. [16] for example) or to implement topology optimization algorithms (see Kook and Jensen [17]). Various ROM strategies have been developed for the WFEM, enabling fast wave analyses at different levels:

- A reduction strategy for computing the forced response for 1D waveguides was developed by Mencik [18,19], along with significant improvements on the computational efficiency and conditioning of the matrices. This yields a better exploitation of the computed wave solutions to retrieve the harmonic response of a finite waveguide.
- Condensation techniques have been introduced in the WFE framework (see Ref. [20]) to reduce the computational effort associated with the dynamic condensation of the inner degrees of freedom (DOF) of the unit-cells.
- Finally, Droz et al. [21,22] developed an interface reduction technique to replace also the periodic edge DOF by a reduced number of propagating Bloch waves. Combined with modal condensation techniques, this method provides the dispersion curves in complex periodic structures with up to 99% reduction of the computational effort.

However, it is emphasized that only propagating and slightly decaying wave solutions can be derived from the wave expansion strategy mentioned above. Nonetheless, evanescent waves are crucial to describe the local dynamics at a waveguide's edges, or close to structural singularities (e.g. a coupling element, a point force excitation). Therefore it is obvious that none of the above mentioned ROM strategies allows both an ultrafast computation of the dispersion curves and a further exploitation for coupling or forced response computation. This work therefore aims at the development of a reduced formulation of the dispersion problem allowing the computation of both propagating and evanescent waves. This will enable a further use of the obtained solutions to predict the response to complex load cases or to perform fast diffusion analyses through finely meshed sub-structures.

In this paper a model order reduction strategy is developed for the wave/finite element framework, based on singular value decomposition of a discretized wave solution subset. The paper is organized into 6 sections including this introduction. The background on WFEM is reviewed in Section 2 and some numerical issues related to Bloch wave analysis are discussed. In Section 3 the proposed model order reduction scheme is described for the fast computation of propagating and evanescent waves based on singular value decomposition. In Section 4, the reduction scheme is applied to a hollow beam, which is a typical case of continuous waveguide exhibiting multimodal behaviour when subjected to a point force excitation in the medium frequency range. The reduced model is used to compute wave dispersion characteristics and the approximation error. Then, the forced response is computed in the finite structure, to highlight the method's performances in the forced WFEM framework. In Section 5, the performances of the method are challenged for a full-scaled stiffened plate with an large unit-cell's FE model. The vibroacoustic behaviour of this periodic structure is commonly studied though wave-based approaches and requires the combination of the WFEM with a CMS technique. The proposed reduction scheme is therefore applied to explore veering, locking and stopband effects produced by the stiffeners, with a frequency resolution that could not be achieved without ROM strategy. Conclusion are eventually drawn in Section 6.

2. Review of the WFEM framework

The WFEM framework correspond to the application of Floquet-Bloch conditions on FE models of unit cells to compute the properties of waves in a periodic media. First, a finite element model is obtained using an FE package, the mesh of the unit cell should respect periodicity conditions ensuring that primal assembly of the right and left interfaces of the unit cell is possible. Then, the degrees of freedom of the unit cell are separated in three groups: q_L the degrees of freedom of the left interface, q_I, the internal degrees of freedom of the unit cell and q_R the degrees of freedom of the right interface. Finally, Bloch periodicity conditions are applied on the unit cell namely: $q_R = \lambda q_L$ and $f_R = -\lambda f_L$. f_L and f_R representing the efforts on the right and left interface and λ being the Floquet-Bloch propagation constant.

2.1. Inverse and direct approach pros and cons

Initially developed by Mead [23], the inverse approach consist in fixing the propagative constant λ of a periodic structure and compute the unknown frequencies ω_i, solutions of the following eigenvalue problem:

$$(\mathbb{K}(\lambda) + j\omega_i\mathbb{C}(\lambda) - \omega_i^2\mathbb{M}(\lambda))\mathbf{\Phi}_i = \mathbf{0} \quad (1)$$

where the matrices $\mathbb{K}$, $\mathbb{C}$ and $\mathbb{M}$ are respectively the stiffness, damping and mass operators of the waveguide's periodic unit-cell obtained by forcing the value of the propagation constant (see Eq. (6) for explicit formulation). It is therefore emphasized that viscous and hysteretic damping models are handled by this formulation, although more complex damping models would require the resolution of non-linear eigenvalue problems:

$$(\mathbf{K}(\omega_i, \lambda) - \omega_i^2\mathbb{M}(\lambda))\mathbf{\Phi}_i = \mathbf{0} \quad (2)$$

The formulation in Eq. (1) has the advantage that the matrices $\mathbb{K}(\lambda)$, $\mathbb{C}(\lambda)$ and $\mathbb{M}(\lambda)$ are symmetric positive whenever the propagating constant λ is equal to 1 or -1 (the solutions ω_i are therefore corresponding to the so-called *cut-on* frequencies). Additionally, these matrices are self-adjoint for all the complex propagating constants located in the unit circle. This means the eigenvalue problem is well-conditioned, especially when no damping is present. The eigenvalues ω_i and eigenvectors $\mathbf{\Phi}_i$ pairs obtained by this method exhibit higher accuracy and reliability, while iterative solvers can be used to limit the computation to a solution subset of interest. This inverse method however, has a number of drawbacks:

a The size of the eigenvalue problem is $(n+m)$ where n is the number of DOF on the section and m is the number of DOF on the inner part of the structure.
b Since frequency is the unknown of eigenvalue problem Eq. (1), the use of frequency-dependent material properties (such as damping) is not straightforward. As a consequence, the prediction of non-propagating waves in dissipative materials is limited.
c The method is not compatible with frequency-based approaches such as the forced/coupling formulations derived from the WFEM outputs (see Mencik and al. [18])

As a consequence of these issues, the direct approach was later developed [24]. The direct approach assumes a known frequency and uses dynamic condensation of the m inner DOF of the structure to reduce the size of the spectral problem from $(n+m)$ to n. An $(n \times n)$ eigenvalue problem is then solved with the propagating constants as unknowns. This eigenvalue problem can take the following among many forms([25,26]):

$$(\frac{1}{\lambda}\widetilde{D}_{RL} + (\widetilde{D}_{RR} + \widetilde{D}_{LL}) + \lambda\widetilde{D}_{LR})q_L = 0 \quad (3)$$