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Multi-inclusions modeling by adaptive XIGA based on LR B-splines and multiple level sets

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ABSTRACT

In this paper, we present an effective computational approach that combines an adaptive extended isogeometric analysis (XIGA) method with locally refined (LR) B-splines and level set methods for modeling multiple inclusions in two-dimensional (2D) elasticity problems. The advantage of XIGA is to model inclusions without considering internal inclusion interfaces by additional functions. Multiple level set functions are used to represent the location of inclusion interfaces and to define enrichment functions. Local refinement for adaptive XIGA using LR B-splines is based on the posterior error estimator. We use the strategy of structured mesh refinement to implement local refinement in adaptive XIGA. Numerical experiments for multiple inclusions with complicated geometries are presented to demonstrate the accuracy and performance of the proposed approach. In addition, numerical results indicate that the adaptive XIGA with local refinement achieves faster convergence rate than that of the XIGA with uniform global refinement.

1. Introduction

Defects in composite materials such as inclusions, voids or cracks are of critical issues and central importance for the structural integrity and durability of components. For instance, material interfaces in multiphase materials are usually taken into account in modeling to predict mechanical behavior and to establish macroscopic material properties. The accurate modeling of voids and inclusions is hence essential for realistic responses, which require not only an appropriate mechanical model but also a sophisticated representation of the geometry. In finite element analysis (FEA), the geometry is modeled normally by an adequate mesh of the microstructure. Despite of well established mesh generation, the generation of conforming meshes is still a time-consuming and burdensome task for complex microstructure geometries. In addition, the accuracy of the finite element method cannot be retained at the interface since the solution is only C^0-continuous on the material interface, which originates from the difference of material properties on the two sides of the interface.

To solve the class of problems with material interfaces where the discontinuity occurs in the strain field, many methods have thus been introduced in the literature. Among them, the extended finite element (XFEM), see e.g. [1,2], and references therein, is one of the effective methods. The basic idea behind the XFEM is to include additional known functions into finite element solution space by means of the partition of unity (PU) [3]. A major advantage in this strategy is to describe the interfaces through a level set function, with an enriched approximation of the finite element scheme to accurately model the different jumps at the interfaces. Sukumar et al. [4] was the first who applied the XFEM integrated with the level set method to model holes and inclusions without containing the interior inclusion interfaces. Yu and Bui [5,6] have recently presented an adaptive scheme in terms of the XFEM for simulation of two-dimensional (2D) weak and strong discontinuities. Their 2D approach was extended to 3D cases, i.e., for

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inclusion problems [7] and cracks analysis [8]. Recently, Tran et al. [9] used a multiple level set approach to prevent numerical artefacts in complex microstructures with nearby inclusions within XFEM. Tornabene et al. [10] presented some numerical applications of composite materials modeled using the well-known Cosserat model, and they used an advanced strong form pseudo-spectral method to deal with geometric and material discontinuities. Fantuzzi [11] solved inclusion problems using the strong formulation finite element method (SFEM), and the SFEM observes fast accuracy and the results are in good agreement with the reference solutions. Dimitri et al. [12] numerically studied the cracking process of bi-material interfaces by combining the XFEM and the level set method (LSM), and remarkable agreements are achieved between the XFEM and the SFEM results.

Isogeometric analysis (IGA) developed by Hughes et al. [13] aims to unify the fields of Computer Aided Design (CAD) and FEA. The principle of IGA is to adopt CAD basis functions (e.g., NURBS, T-spline) as shape functions of FEA, the IGA thus possesses many good properties such as the exactness of reproducing the geometry, higher-order continuity, simple mesh refinement, and avoiding the traditional mesh generation procedure. The IGA has been successfully applied in many areas of engineering and science, see e.g., [14–20]. The authors recently have used the IGA to solve some problems [21–28]. Based upon the idea of enrichment concepts, the IGA has been enhanced by adding appropriate enrichment functions in the approximation spaces for problems with discontinuous solutions, which forms a similar discrete discretization scheme of the XFEM. However, the extended isogeometric analysis (XIGA) takes all the advantages of XFEM that can model the problems with discontinuous solutions without considering the interior discontinuous interfaces. Many researchers and scientists have further extended and applied the XIGA to solve a class of discontinuities problems for different engineering materials. For instance, Luycker et al. [29] and Ghorashi et al. [30] estimated stress intensity factors and crack propagation in 2D elasticity. Bui [31] studied dynamic and static crack behaviors in smart piezoelectric materials. Bayesteh et al. [32] analyzed thermal-mechanical fracture of inhomogeneous cracked functional materials. The XIGA was applied to address weakly discontinuous problems by Jia et al. [33]. Recently, Singh et al. [34] proposed a simple, efficient and accurate Bézier extraction based T-spline XIGA for crack simulations, and Nguyen et al. [35] presented an adaptive XIGA based on polynomial splines over hierarchical T-meshes (PHT-splines) for modeling crack propagation. Consequently, the XIGA has been shown to be a powerful technique for numerical simulation for a wide range of discontinuity problems. On the other hand, in IGA context, B-splines and non-uniform rational B-splines (NURBS) [36,37] are the most widely used as its basis functions. Because of their high order continuity, the implementation of B-splines or NURBS in IGA gives a high order continuous approximation. However, the lack of local refinement ability makes both B-splines and NURBS a significant challenge to be used in an effective IGA since they are characteristically formulated as global tensor products of several univariate B-splines. The situation becomes more critical once modeling discontinuous problems, where several local regions crucially require fine meshes. To overcome their limitations, several alternative splines that offer local adaptive refinement have been proposed and investigated, such as T-splines [38–40], truncated hierarchical B-splines [41], PHT-splines [42,43], and locally refined (LR) B-splines [44]. Among them, LR B-splines, which was first used in adaptive IGA by Johannessen et al. [45], provide more versatile choices for refinement strategies [46], and thereby, have emerged as potentially alternative framework in IGA. Kumar et al. [47] then presented a posteriori error estimation technique in adaptive IGA using LR B-splines. In addition, LR B-splines have been applied to solve several problems in fluid mechanics [48,49]. On the other hand, since the XIGA is established based on the framework of IGA, it is highly potential to implement the LR B-splines into XIGA to develop an efficient method for modeling internal interfaces. However, to the best of our knowledge, there is still no such adaptive XIGA implementation for inclusions available in the literature.

In this paper, we develop an effective computational approach that combines an adaptive XIGA method using LR B-splines and multiple level set method for modeling multiple inclusions in 2D elasticity without meshing the internal interfaces. Inclusions are represented and formulated through a coupling setting between XIGA and the level set method [50]. Particularly, multiple level set functions are used to represent inclusion interfaces and applied to define enrichment functions. According to Zienkiewicz and Zhu method [51], the strain recovery technique is developed to obtain the posterior error estimator. Local refinement is implemented based on the posterior error estimator and the strategy of structured mesh refinement [45]. The desirable characteristics of the developed adaptive approach are illustrated through six numerical experiments with single and multiple inclusions problems. Based on the numerical examples, we will address the efficiency and accuracy of the proposed adaptive XIGA.

The advantages of the proposed methodology are as follows: (1) inclusions modeling using the XIGA does not consider the internal inclusion interfaces compared with the traditional IGA; (2) XIGA based on LR B-splines has the features of the NURBS-based XIGA, however, the present method can be locally refined, which is not available in the NURBS-based XIGA. Thus, the required domain will be refined to improve the accuracy at a low cost; (3) owing to the higher-order continuity of B-spline basis functions, the resulting stresses derived from the present method are smooth, which are not available in the XFEM with C^0-continuity of inter-elements; and the present method has the higher accuracy and higher-order convergence rate over the conventional XFEM.

The paper is organized as follows. Section 2 presents the fundamental equations for inclusions in 2D elasticity. In Section 3, B-splines, NURBS and LR B-splines are briefly introduced. The formulations of XIGA for multiple inclusions are presented in details. In addition, the posterior error estimator based on strain recovery is provided. At the last of the section we offer a procedure of adaptive XIGA method for multiple inclusions. Six numerical examples are presented in Section 4 to verify the efficiency and accuracy of the proposed method. The paper ends with a summary in the last section.

2. Fundamental equations

Consider a body occupying an open bounded domain $\Omega \in \mathbb{R}^2$, with boundary Γ. The domain Ω is composed of multiple homogeneous isotropic materials. The boundary $\Gamma = \Gamma_u \cup \Gamma_t \cup_{k=1}^{N_{ints}} \Gamma_I^k$, where Γ_u, Γ_t are the Direachlet displacement and Neumann traction boundary, respectively, Γ_I^k are the material interfaces and N_{ints} is the total number of interior material interfaces. Traction is continuous along the material interfaces Γ_I^k. The mechanical equilibrium equations of linear elastostatic problems are given by:

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0} \quad \text{in } \Omega, \tag{1a}$$

$$\boldsymbol{u} = \overline{\boldsymbol{u}} \quad \text{on } \Gamma_u, \tag{1b}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \overline{\boldsymbol{t}} \quad \text{on } \Gamma_t, \tag{1c}$$

$$[\![\boldsymbol{\sigma} \cdot \boldsymbol{n}_I^k]\!] = 0 \quad \text{on } \Gamma_I^k, \quad k = 1, \ldots, N_{ints}, \tag{1d}$$

where $\boldsymbol{u}$ is unknown displacement field; $\boldsymbol{\sigma}$ is the stress tensor; $\boldsymbol{b}$ is the body force; $\overline{\boldsymbol{u}}$ and $\overline{\boldsymbol{t}}$ are the prescribed displacement and traction boundary conditions, respectively; and $\boldsymbol{n}$ is the unit outward normal to Ω. The continuity of tractions along the material interfaces is guaranteed by the last equation.

The strain and stress fields are expressed as

$$\boldsymbol{\varepsilon} = \nabla_s \boldsymbol{u}, \tag{2a}$$