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Transient wave scattering and forced response analysis of damaged composite beams through a hybrid finite element-wave based method

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ABSTRACT

A new numerical method for the waves-damage interaction and the post-failure response analysis of laminated composite beams under an impact load is presented in this paper.
The studied structure is made of E-glass/epoxy and it is decomposed in undamaged and damaged substructures. The material properties in the damaged substructure are modified according to sudden material property degradation rule. The damage describes the creation of micro-discontinuities in the structure during the loading. The enhanced formulation considered in this paper can be used to target this problem. The numerical method combines the Finite Element (FE) method to model the discontinuous regions, impact region and damaged region, and the wave finite element (WFE) to accurately model the periodic substructures (intact substructures). This hybrid method enables the calculation of the waves-damage interaction scattering coefficients and the response of the whole structure in both the frequency as well as in the time domain and it provides the reduction of the computational cost of analyses in space and time.
Several numerical simulations have been performed, first to validate the accuracy of the results predicted by the proposed formulation and second to investigate the damage effect on the dynamic behaviour of the structure.

1. Introduction

For several years, composite materials are widely used in the realization of structures in many industrial fields, such as aerospace and automotive, due to their superior physical and mechanical properties over conventional engineering materials. Despite these high specific mechanical properties, composites are fragile and sensitive to damage from a number of sources, both during manufacturing and service.

Due to their anisotropic nature, composite structures may exhibit a various forms of damage [1], such as matrix cracks, delamination and fibre breakage, which can cause considerable degradation of the mechanical properties and consequently which can change, locally, its mechanical behavior. These modifications can induce a significant reduction in the structure lifetime. Since it is not possible to avoid the damage in the structure lifetime, it is necessary to estimate its behavior in service if the mechanical characteristics are degraded in order to ensure the reliability and safety of designs.

Therefore, it is very important to determine the mechanical parameters that are degraded for each failure mode. The effective elastic constants of a laminate containing defects can be adequately predicted by reducing the appropriate elastic properties of the damaged ply according to the material degradation models [2,3]. These models are mathematical representations that properly define the residual stiffness of the damaged material.

To date, many material degradation models have been proposed and classified as two categories, sudden degradation models and gradual degradation models [4–7].

Due to their simplicity, satisfactory physical and computational efficiency, the majority of failure analyses of composite structures are performed using the sudden degradation rules. The material properties associated with failure modes are reduced, instantaneously, to zero [8,9] or to some proportion of the undamaged properties [10–12], which is called a degradation factors. These degradation factors were determined by empirical observation and correlation with experimental results. Shokrie and Tserpes [8,9] assumed that partial material prop-

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erties associated with failure modes have been decreased completely to zero for each failure mode, including the failure of fiber, matrix and delamination. In practice, small values are maintained in place of zero to avoid the computational problems induced by degrading the properties completely to zero values.

Tan [11] presented a degradation model for composite laminates containing opening under in-plane loading. This degradation model was based on parametric studies and experimental results. Camanho and Matthews [10] proposed a material degradation model as an extension of Tan's model to 3D problems. This model distinguishes between the tensile and compression failure modes both for the fiber and matrix. Zhang et al. [12] developed a theoretical material degradation model based on micromechanics. This model provides a generalized method to estimate the degradation factors from the fundamental properties of composite materials.

The damage describes the development of micro-discontinuities in the structure during the loading. Therefore, it is necessary to analyse the scattering induced by discontinuities in the structure. Over the last decades, many numerical models for analysing the behaviour of damaged composites were extensively investigated, including the finite element method (FEM) [7,13–15], an enriched finite element method (PNM-GFEM) [16] and spectral finite element method (SFEM) [17–20].

Finite Element analysis is the most popular numerical tool to analyse the wave propagation in discontinuous domains. However, the high numerical costs and the excessive computational time related to large models in mid and high-frequency ranges, constitute one of the major limitations of this method.

The Wave Finite Element (WFE) method [21–24] can be an alternative which provide a large decrease of time and cost for obtaining the forced responses of systems compared to the conventional finite element method.

This approach is based on the finite element model and periodic structure theory which starts from a FE model of only a typical substructure [25] of the periodic structure, then the mass and stiffness matrices of this substructure are used to formulate an eigenvalue problem whose solutions yields the wave properties (wavenumber, wave modes, group velocity, modal density, etc.).

In recent years, it has been extensively proposed to simulate free [26] and forced [27] wave propagation behaviors in periodic structures with different natures, such as beam-like structures [28], laminated beams [29] and plates [26], fluid filled pipes [30,31], etc. The forced response of a structure can be predicted by this wave approach if the structure is periodic. If the structure is containing discontinuities, is partly periodic, wave approach can still be applied using the diffusion matrix method [32] or hybrid WFEM/FEM [33,34]. The damaged substructure is modeled by conventional Finite Element Method while the intact substructures are regarded as waveguides and modeled using the Wave and Finite Element Method to reduce the dimension of problem and accelerate the computing.

In this paper, a FE/WFE hybrid method is developed to determine the waves-damage interaction scattering coefficients then the forced response of damaged laminated composite structures. A new Diffusion Matrix Formulation is proposed to analyse the scattering induced by discontinuities in structure. The Wave finite element approach is formulated in the frequency domain and therefore the Inverse Discrete Fourier Transformation of the frequency response to describe the dynamics in time domain, is required.

The structure studied is made of E-glass/epoxy and it is decomposed in undamaged and damaged substructures. The damaged substructure is characterized by the reduction of the material properties according to sudden material property degradation rule. Therefore, wave propagation in damaged structures can be analyzed exactly using this enhanced formulation during the loading.

The remainder of this paper is organized as follows. An overview of the WFE method and the extension of this method in time domain is presented in Section 2. Next, the enhanced hybrid WFE/FE method is formulated in Section 3. The scattering matrix formulation is presented, and the forced response is expressed. The Material Property Degradation model is described in Section 4. In Section 5, Some numerical examples will be provided. Finally, concluding remarks and perspectives of this work are given in Section 6.

2. Wave finite element method in time domain

The WFE method is a spectral analysis method, classically formulated in the frequency domain, which is described in detail in Refs. [21,33,35]. Here, the application of this method to obtain also the time-domain responses via the Inverse Discrete Fourier Transform (IDFT) is described. The main steps to analyse the free waves and forced response by this approach are summarized in Fig. 1:

- Step 1: The first step of this method is to transform the excitation force F_{ext}, sampled into M point signal time $[t_k]_{(k=1..M)}$, into the frequency-domain via Discrete Fourier Transform (DFT).
  The spectrum of this excitation force $\widetilde{F}_{ext}$ can be expressed in the frequency-domain $[\omega_k]_{(k=1..M)}$:

$$\widetilde{F}_{ext}(\omega_k) = \sum_{m=1}^{M} F_{ext}(t_m) e^{-jt_m\omega_k} \tag{1}$$

- Step 2: Next, this spectrum is used in the WFE approach to calculate the nodal displacement response at each discrete frequency. The formulation of the WFEM in the frequency domain is briefly recalled.
  Consider a periodic structure composed of N identical connected substructures along axis x.
  The WFE method is based on the finite element model of a typical substructure of length d belonging to the global structural waveguide using a FE software. Assume that the left and right boundaries of the discretized substructure contain the same number of degrees

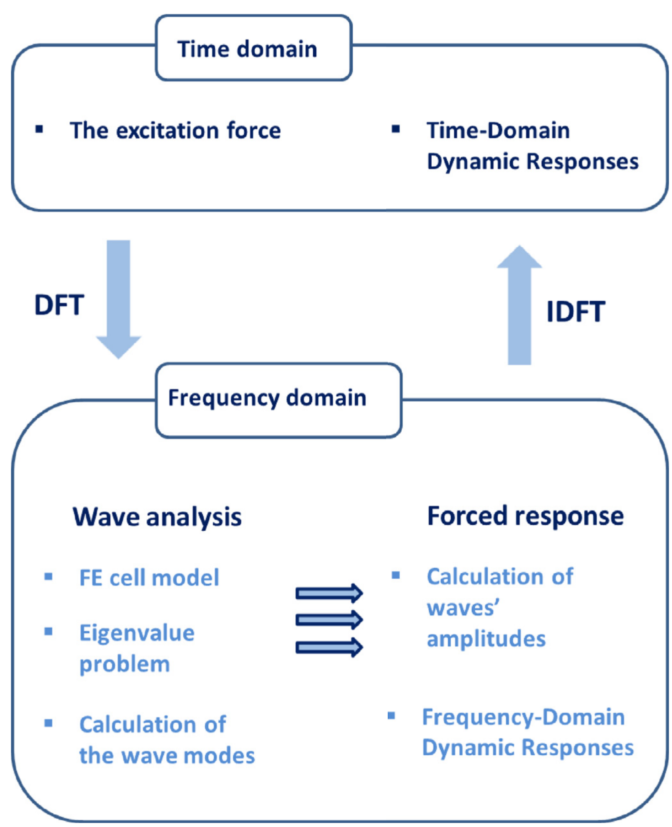

Fig. 1. Analysis procedure of the WFEM in Time-Domain.