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Elastic stability of curved nanobeam based on higher-order shear deformation theory and nonlocal analysis by finite element approach

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ABSTRACT

In the present work, elastic stability analysis of curved nanobeams is investigated using the differential constitutive law consequent to Eringen's strain-driven integral model coupled with a higher-order shear deformation theory accounting for through thickness stretching effect. The formulation developed here is general in the sense that it can be deduced to realise the influence of different structural theories and analyses of nanobeams. The governing equations derived are solved employing finite element method using a 3-nodes curved beam element. The model developed here is validated considering problems for which analytical/numerical solutions are available in the literature. For comparison purpose, results are also presented for various structural theories obtained from the present formulation. The influence of structural and material parameters such as thickness ratio, beam length, rise of the curved beam, boundary conditions, and size-dependent or nonlocal parameter are brought out on the buckling behaviours of curved nanobeams. It is observed that the type of buckling mode pertaining to the lowest critical value can be different depending on geometrical and internal material length scale parameter, and boundary conditions.

1. Introduction

There is a great demand in introducing nanomaterials because of their excellent properties in the development of micro/nano electromechanical systems (MEMS/NEMS). Furthermore, the structural elements like nanobeams, plates and shells can be part of such systems in nanotechnology and its applications. Therefore, understanding the characteristics of nanostructural elements through experiment and/or mathematical modelling and simulation is of utmost importance for the efficient design of nanosystems and such studies gain importance among researchers. However, the modelling based on the classical continuum theories may not bring out the nano-structural responses due to the absence of internal material length scale in the constitutive relation. Although experiments and mathematical models based on atomistic approaches can capture the size dependant effects, these attempts are rather tedious and computationally expensive. Hence, the numerical simulation of nanostructures employing size-dependent continuum theories has been largely attempted in analysing the response behaviours of nanostructures under different environment.

The behaviours of nanostructures are investigated widely using size-dependent continuum mechanics models based on the Eringen nonlocal theory of elasticity as given in the work of Eringen [1,2] and Eringen and Edelen [3]. The Eringen nonlocal model was formulated employing the integral constitutive equation describing the dependence of the stress at a point on the strain in the rest of the domain through a positive-decaying kernel function. It is further simplified in the sense that the nonlocal integral constitutive equation is transformed into a differential form for a specific class of kernel functions, thus leading to much easier to handle than the integral model. From the work of Peddieson et al. [4] and due to this simplicity, this differential Eringen nonlocal model has been widely employed to investigate the static, buckling, and dynamic characteristics of nanostructures. Although the available research work using the Eringen nonlocal model is exhaustive, for the sake of brevity, some of the important contribution detailed here are for static bending [5–13], buckling [13–24] and vibration responses [13,19,25–30]. These studies are made using Euler-Bernoulli theory by Wang and Liew [9], Ngoc-Tuan Nguyen et al. [11], Senthilkumar [15], Wang et al. [9], Wang and Varadan [27], Zhang et al. [28], Murmu & Pradhan [29] and recently Taghizadeh et al. [20]. Timoshenko theory

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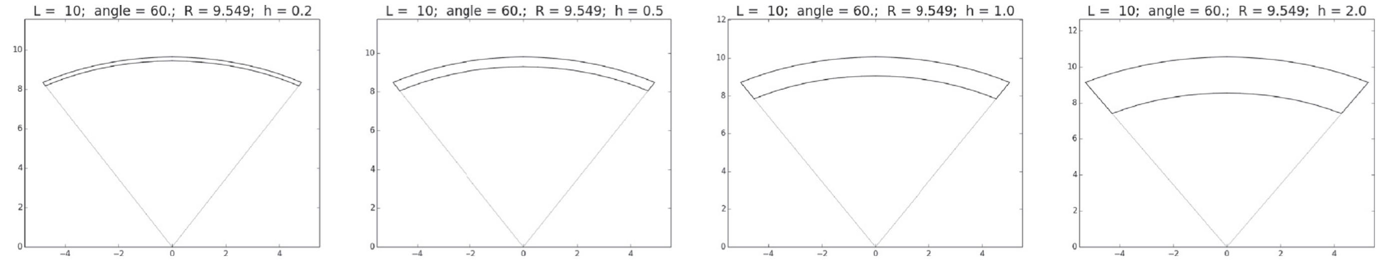

Fig. 1. Geometry of curved beams of constant length-to-radius ratio L/R: from thin to thick.

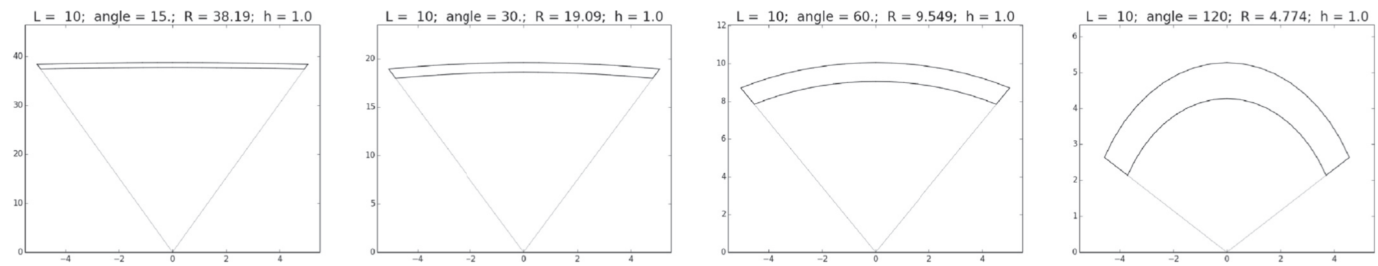

Fig. 2. Geometry of curved beams of constant span-to-thickness ratio L/h: from shallow to deep.

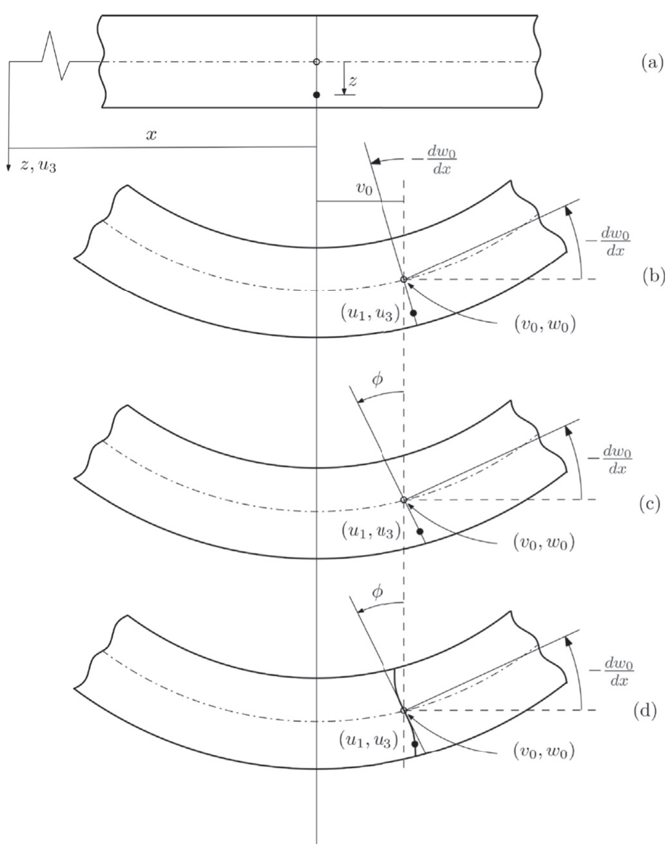

Fig. 3. Undeformed and deformed geometry of a beam section: (a) undeformed; (b) Euler-Bernoulli; (c) Timoshenko; (d) higher order theories.

is considered in the work of Reddy and Pang [7], Reddy [8], Roque et al. [10], Pradhan and Mandal [12], Thai [13], Senthilkumar et al. [16], Challamel [19], Janghorban and Zare [26], Murmu and Pradhan [29], and Wang et al. [21,30]. The higher-order beam theory is assumed by Reddy [6], Thai [13], Ansari and Sahmani [14], Challamel [19], Aydogdu [25], Eltaher et al. [22], Rahmani and Jandaghian [23], and Emam [24].

The Eringen nonlocal analysis is also extended to analyse nanoplates [31–37]. The bending analysis of nanoplates is examined by Aghababaei and Reddy [31], Phadikar and Pradhan [32], yan et al. [33], and Nguyen et al. [34] whereas buckling study is investigated in Refs. [32,34] and by Pradhan [35], and Analooei et al. [36]. The vibration of nanoplates is highlighted in Refs. [31–34,36], and by Pradhan and Phadikar [37], Malekzadeh et al. [38], Ansari et al. [39], Pradhan and Kumar [40], and Shahidi et al. [41]. These work are based on classical theory [31–33,36,40,41], first-order theory [31,38,39], and higher-order theory [31,35,37]. The investigation on curved nanobeams and nanoshell structures is also examined in the literature [42–47]. Dynamic characteristics of curved nanobeam is carried out analytically using Timoshenko beam theory by Hosseini and Rahmani [42] whereas higher-order theory is introduced in the work of Ganapathi and Polit [43]. Hu et al. [44], Li and Kardomateas [45], and Arash and Ansari [46] attempted to investigate the problems using classical theory whereas Wang and Varadan [47] dealt with first-order shear deformation theory. These work were focused to understand the bucking [45], vibration [46], and wave propagation [44,47] characteristics of nanoshells. A review on the modelling of carbon nanotubes and graphenes is presented by Arash and Wang [48] and most recently by Eltaher et al. [49]. Recent studies on certain class of problems, for instance bending of cantilever beam, using directly the Eringen integral constitutive equation reveal some inconsistency while solving through the Eringen differential model as brought in the work of Challamel and Wang [50], Challamel et al. [51] and also while dealing with a bar in tension by Benvenuti and Simone [52], and more recently by Fernandez-Saez et al. [53], and Romano et al. [54].

It is observed that a large amount of available work deals with the analysis of straight nanobeams and nanoplates. However, the study on

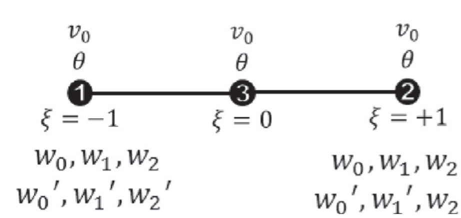

Fig. 4. Beam element with the degrees of freedom per node.