Finite Elements in Analysis and Design 146 (2018) 70–83

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Improved recovered nodal stress for mean-strain finite elements

R. Sivapuram, P. Krysl *

University of California, San Diego, 9500 Gilman Dr., #0085, La Jolla, CA 92093, USA

ARTICLE INFO

Keywords:
Nodal stress
Mean stress
Nearly incompressible
Mean-strain
Quadratic tetrahedron
Hexahedron
Finite element
Stress extrapolation

ABSTRACT

This paper investigates a method for improving the accuracy of the stress predicted from models using the mean-strain finite elements recently proposed by Krysl and collaborators [IJNME 2016, 2017]. In state-of-the-art finite element programs, the stress values at the integration points are commonly post-processed to obtain nodal values of stress. The mean stresses are element-wise constant, and hence the nodal values obtained from the mean stresses tend to be of lower accuracy. The proposed method post-processes the uniform stress in each element in combination with a linearly-varying stabilization stress field to produce a more accurate representation of the nodal stresses. Selected examples are presented to demonstrate improvements achievable with the proposed methodology for hexahedral and quadratic tetrahedral mean-strain finite elements.

1. Introduction

A few recent publications described high-performance mean-strain finite elements based upon the idea that the rank-deficient mean-strain element can be stabilized (in the sense of correcting the rank deficiency) by setting up two forms of stabilization energy that is sampled with the full quadrature rule or with the mean-strain quadrature [1–6]. These elements achieve insensitivity to material constraints (for instance isochoric), and they are applicable to the modeling of thin structures. The mean-strain approach however makes the stress post-processing more challenging. While the stresses are uniform element-wise, the mean-strain elements achieve high accuracy in displacements. Consequently it is reasonable to expect that using the accuracy inherent in the displacement solution, there might be some way of boosting the accuracy of the stresses as well. This is the motivation for the present work.

The stress values in Finite Element Analysis (FEA) are connected to the integration points. A common post-processing operation for stresses in FEA is to recover continuous stress fields from the quadrature-point stresses. In order to visualize the stress distribution, the stress is extrapolated from the quadrature points to the nodes of each element. Then the stress field can be visualized element-wise using filled-contour plots, isosurfaces, etc., but it is (typically) discontinuous at the inter-element boundaries. Alternatively, the nodal stresses can be made unique at each node shared by several finite elements by some form of "averaging" of the element-wise stress predictions at the node. In order for this averaging to work well, the stress predictions at the nodes of each element must be of good quality. This condition is not satisfied when using the mean-strain elements, such as the elements proposed in Refs. [1–6], or the hexahedral elements implemented in the Abaqus solvers [7]. In this work, we attempt to improve accuracy of the integration-point stresses extrapolated to the nodes of an element.

First, let us mention some procedures from literature for extracting nodal quantities from an element. One popular technique for improved stress approximation is the 'superconvergent patch recovery' (SPR) method developed by Ref. [8]. It is developed based on the presence of superconvergent points in a finite element, where the stresses have an order of accuracy higher than rest of the finite element region. The stresses are fitted using a polynomial of one order higher than that of the strains, in a least squares sense. However, the presence of superconvergent points is not always guaranteed, for example, in curved elements. Also, in some element configurations, for instance, elements located at corners or at edges of three-dimensional geometries may not provide enough superconvergent points around a given node to enable the requisite least-squares solution. In this case the SPR, extrapolation fails and needs to be replaced with a simpler, less accurate, procedure.

The nodal point forces in a finite element were used by Refs. [9,10] to compute interpolated stresses which are shown to be enhanced in quality as compared to the directly-computed stresses in triangle, quadrilateral and tetrahedral elements. The stresses at a node are computed using an average over a patch of elements containing the node.

* Corresponding author.
E-mail address: pkrysl@ucsd.edu (P. Krysl).

https://doi.org/10.1016/j.finel.2018.04.005
Received 23 January 2018; Received in revised form 12 April 2018; Accepted 14 April 2018
Available online 3 May 2018
0168-874X/

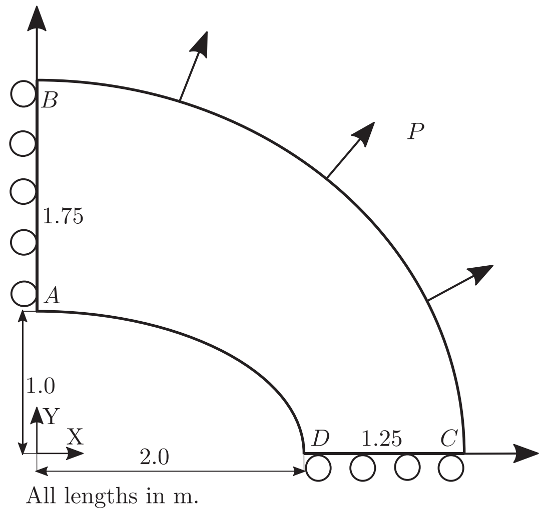

Fig. 1. Elliptic membrane.

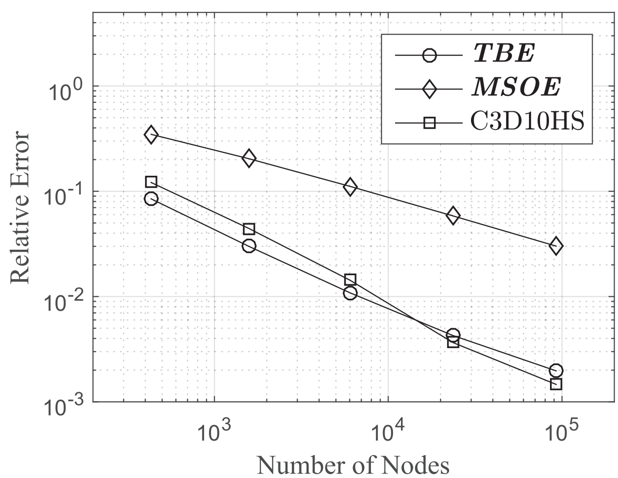

Fig. 2. LE1 benchmark. Errors in σ_{yy} at Point D with mesh refinement (Quadratic Tetrahedral elements).

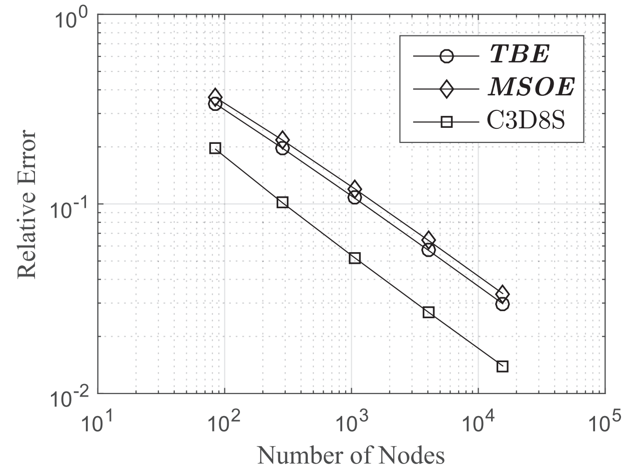

Fig. 3. LE1 benchmark. Errors in σ_{yy} at Point D with mesh refinement (Hexahedral elements).

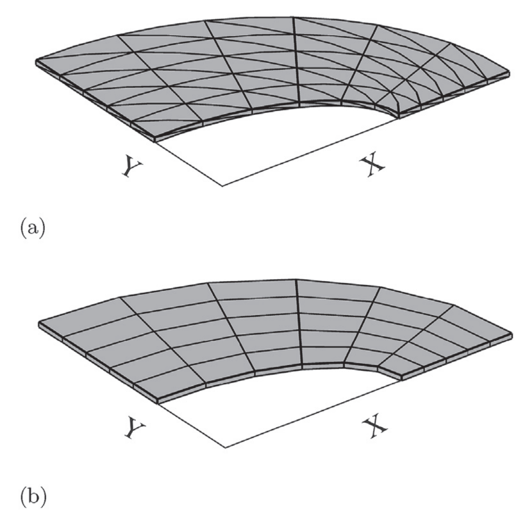

Fig. 4. LE1 Benchmark, Coarsest meshes used for mesh refinement study (a) Quadratic Tetrahedral elements (b) Hexahedral elements.

Since the stresses computed are based on the real material, achieving improved stress approximation in nearly incompressible materials is difficult. An enhanced stress approximation was proposed in Ref. [11] by assuming a richer interpolation space for the stresses and by improving the fulfillment of equilibrium by weakening the equilibrium in a small patch of elements.

The aforementioned stress computation procedures are applicable to the mean-strain finite elements as well. In comparison, the present approach has perhaps the advantage of simplicity, which may translate to a higher computational efficiency. The paper is organized as follows. In Section 2, the mean-strain formulation is derived from a variational principle for linear elasticity to obtain the stiffness matrix generated only by the constant-strain modes [1,3,6]. This can lead to the formation of hourglass modes, and stabilization is required to prevent this. The design of the stabilization material model to suppress the rigid body modes and to represent the bending strain energy accurately is discussed in Section 3. The stabilization strain energy is sampled using two quadrature rules, full quadrature and mean-strain quadrature to prevent the stiffness matrix from being rank-deficient while at the same time guaranteeing convergence. The proposed stress computation procedure which is expected to be improved over the mean stresses is discussed in Section 4. The proposed stress field is derived using the mean-strain theory and stabilization energy concepts, and a linear stress field is computed within each element by extrapolating the stabilization material stresses from the integration points to the nodes of each element. Section 5 presents some selected examples demonstrating the improvement of stress prediction using the proposed stress field over the mean stresses for static, compressible and nearly incompressible material models meshed using quadratic tetrahedral and hexahedral elements. The examples used for verification in this work are an elliptic membrane (Subsection 5.1), thick plate under pressure loading (Subsection 5.2), an infinite slab with a stress-free hole under far-field tensile loading (Subsection 5.3), thin cantilever beam with end shear loading (Subsection 5.4) and fibrous composite cube under general synthetic quadratic displacements (Subsection 5.5). Section 6 summarizes the work and presents the key conclusions.

2. Mean strain formulation

In this section, a brief review of mean-strain finite element formulation for linear elasticity from Refs. [1,3,6] is presented. We confine our presentation to the parts that are essential for developing the proposed stress field. In this work, we consider both the mean strain hexahe-